HOMER COMMUNITY SCHOOOL DISTRICT

Mathematics Curriculum

Implemented 2013/2014

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Mission Statement

Mission Statement

The Homer Community School District provides a safe, supportive environment in which all students will develop the skills, knowledge, and integrity essential for a successful future. This includes educating students to:

- read with understanding,
- communicate clearly,
- solve problems effectively,
- think critically, and
- act responsibly.

Philosophy and Beliefs

Mathematics Education Philosophy and Beliefs

Philosophy

The Homer Community School District believes the math curriculum should meet diverse needs of students to become lifelong problem solvers in an ever changing global society. Students will be exposed to a mathematically rich environment; focusing on number sense, algebra, geometry/measurement, data analysis/probability, which empowers them to apply mathematics in all aspects of their lives

Mathematics Education Program Belief Statements

The Homer Community School District believes that:

- All students can learn mathematics and should be held to high expectations.
- All students deserve an excellent program of instruction in mathematics that challenges each student.
- Quality mathematics education is enhanced through community and home support.
- All students will be provided with access to interventions and enrichment opportunities.
- When students take ownership and are able to explain their thinking, math becomes relevant.
- Computational skills and number concepts are essential components of the mathematics curriculum.
- Learning mathematics is an active, collaborative process.
- All students should be able to use appropriate technology and tools to investigate mathematical concepts to increase their understanding.
- Assessment supports the learning of mathematics and provides useful information. Assessments should be varied, on-going, and embedded into instruction.
- On-going professional development is a key component of an effective mathematics program.

Research and Program Model

Review of Literature

The members of the Mathematics Curriculum Review Committee examined research on effective mathematics instruction and to find out what kinds of programs or strategies significantly impacted student achievement. The following information is a summary of information obtained in that query.

The National Council of Teachers of Mathematics publishes a wealth of information for teachers and schools in mathematics. In *Principles and Standards for School Mathematics* (2000), four major components are essential in a mathematics program. First, the principles for school mathematics reflect basic perspectives on which educators should base decisions that affect school mathematics. These principles establish a foundation for school mathematics programs by considering the broad issues of equity, curriculum, teaching, learning, assessment, and technology.

NCTM's Six Principles

One of the most essential features of *Principles and Standards for School Mathematics* (2000) is the articulation of six principles essential to high-quality mathematics education. According to the standard, the principles must be “deeply intertwined with school mathematics programs” (NCTM, 2000, p. 12). The principles make it clear that excellence in mathematics education involves much more than simply listing content objectives. The principles include:

- The Equity Principle. “Excellence in mathematics education requires equity-high expectations and strong support for all students” (NCTM, 2000, p. 12).

- The Curriculum Principle. : “A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades” (NCTM, 2000, p. 14).

- The Teaching Principle. “Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (NCTM, 2000, p. 16).
- The Learning Principle. “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM, 2000, p. 20).

- The Assessment Principle. “Assessment should support the learning of important mathematics and furnish useful information to both teachers and students” (NCTM, 2000, p. 22).

- The Technology Principle. “Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning (NCTM, 2000, p. 24).

The Effective Mathematics Classroom

According to Checkly, (1997); Wood and Sellars, (1996); and Wood & Sellars, (1997); the teaching of mathematical concepts and should be structured around problems to be solved.

Johnson and Johnson, (1975) and Davidson, (1990) found positive effects students when students worked cooperatively with others.

An effective classroom uses group problem solving to stimulate students to engage in mathematical thinking (Artzt, Armour-Thomas, & Curcio, 2008).

Activities should be structured around allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999).

Three critical components of effective mathematical instruction include teaching for conceptual understanding, developing children’s procedural literacy, and promoting strategic competence through meaningful problem solving investigations (Shellard & Moyer, 2002).

Students in the middle grades are at an important point in their mathematical education (Protheroe, 2007). They are forming opinions of their mathematical abilities, interests, and motivation, which affect how they approach mathematics in later years. Instruction for these learners should call for increasingly abstract reasoning, hypothetical thinking, and reasoning in both concrete and abstract terms.

The teacher should be engaged in demonstrating acceptance of students’ divergent ideas, challenging students to deeper thinking, allowing students to explain solutions to problems and how they arrived at them, posing challenging and interesting questions to encourage students’ thinking, and projecting a positive attitude towards mathematics ad about students’ ability to be successful at mathematics (Protheroe, 2007).

Students should be actively engaged in mathematics, not just watching others. They should solve challenging problems, from the real world applications, and make interdisciplinary connection to art, architecture, literature, health, and science. Students should share mathematical ideas in pairs, small groups, and class presentations. They should learn to use multiple representations to communicate mathematical ideas, leaning to use models and manipulatives to solve problems that are aligned with the lesson’s objectives (Protheroe, 2007).

Education Alliance (2006) concluded that that the following instructional practices promoted a deeper understanding of mathematics:

- Lessons focused on concepts and skills that are standards-based.
- Differentiated instruction using flexible grouping, individualized lessons, compacting, using tiered assignments, and varying question levels.
- Instructional activities that are learner-centered and emphasize inquiry and problem solving.
- Use of experience and prior knowledge as a basis for new learning.
- Use of cooperative learning and real-life connections.
- Use of scaffolding to help students make connections to concepts, procedures, and understanding.
- Asking probing questions which require students to justify their answers.
- Emphasis on the development of basic computation skills.

Curriculum

- Should be comprised of challenging content
- Standards-based
- Clearly identify the concepts and knowledge to be mastered

Teachers should be provided professional development which focuses on the following ideas:

- Knowing and understanding the standards
- Using the standards as the basis for instructional planning
- Teaching using best practices
- Multiple approaches to assessment
- Using instructional technology tools

Additionally, effective professional development

- Incorporates instructional support materials such as curriculum maps and pacing guides
- Establishes math leadership teams and provide math coaches

Technology

- Students should have access to a variety of technology tools
- Teachers should integrate technology into all mathematics instruction and courses.

Manipulatives

- Manipulatives are an effective instructional tool
- Should be aligned with math concepts
- Help students understand word problems
- Help students develop understanding of mathematical concepts

Assessment

- Should be aligned with content being taught
- Should be used to evaluate student performance and teacher effectiveness
- Should utilize student self-monitoring techniques

- Provides guided practice with feedback
- Utilized for error analyses of student work
- Should incorporate traditional and alternative assessment strategies
- Includes formative, summative, and diagnostic strategies
- Should progress toward increasing the use of open ended assessment techniques

The National Center for Educational Achievement (2009) found that in higher performing middle and high schools:

- Students have a high level of engagement
- Teachers demand higher-order thinking
- Teachers use an inquiry-based instruction that includes cooperative learning, direct instruction, labs, hands-on investigation, and manipulatives.
- Teachers connect to students' prior knowledge and help them make meaningful real-world applications.
- Teachers integrate literacy activities into the courses—including content-based reading strategies and academic vocabulary development.
- Teachers foster an environment where students feel safe trying to answer questions, make presentations, do experiments, and be allowed to make mistakes.

Slavin, Lake, and Groff (2010) studied the effects of mathematics instruction and found that cooperative learning, classroom management, and motivation programs have larger impacts on student achievement than programs that emphasize textbooks or technology alone, and that the most successful math programs encourage student interaction. They found very little evidence that it mattered which curriculum was used when looking at standardized test scores. They found that computer-assisted instruction can help to identify elementary students' strengths and weaknesses, and provide exercises to strengthen weak areas. There was not enough evidence to recommend one program over another, but the outcomes were stronger for computation than for concepts of problem solving. Findings for middle- and high-school students showed little evidence of effectiveness. One limitation was that there are new programs coming out with new technologies, and further studies would need to be done to examine new technologies.

Research-based Top Ten Strategies for Mathematics Achievement

The following instructional practices are cited as being the top ten research-based strategies for mathematics achievement: "Improving Student Achievement in Mathematics: Part 1: Research Findings", by Douglas A. Grouws & Kristin J. Cebulla; December 2000 (Updated January 2002). Published by ERIC 1.

1. Opportunity to Learn
2. Focus on Meaning

3. Problem Solving
4. Opportunities for Invention and Practice
5. Openness to Student Solutions and Student Interactions
6. Small Group Learning
7. Whole-Class Discussion
8. Focus on Number Sense
9. Use Concrete Materials
10. Use Calculators

1. Opportunity to Learn

The extent of the students' opportunity to learn mathematics content bears directly and decisively on student mathematics achievement. Opportunity to learn (OTL) was studied in the First International Mathematics Study (Husén, 1967), where teachers were asked to rate the extent of student exposure to particular mathematical concepts and skills. Strong correlations were found between OTL scores and mean student achievement scores, with high OTL scores associated with high achievement. The link was also found in subsequent international studies, such as the Second International Mathematics Study (McKnight et al., 1987) and the Third International Mathematics and Science study (TIMSS) (Schmidt, McKnight, & Raizen, 1997).

Classroom Implications:
It seems prudent to allocate sufficient time for mathematics instruction at every grade level. Short class periods in mathematics, instituted for whatever practical or philosophical reason, should be seriously questioned. Of special concern are the 30-35 minute class periods for mathematics being implemented in some middle schools.

Textbooks that devote major attention to review and that address little new content each year should be avoided, or their use should be heavily supplemented. Teachers should use textbooks as just one instructional tool among many, rather than feel duty-bound to go through the textbook on a one-section-per-day basis.

It is important to note that opportunity to learn is related to equity issues. Some educational practices differentially affect particular groups of students' opportunity to learn. For example, a recent American Association of University of Women study (1998) showed that boys' and girls' use of technology is markedly different. Girls take fewer computer science and computer design courses than do boys. Furthermore, boys often use computers to program and solve problems, whereas girls tend to use the computer primarily as a word processor. As technology is used in the mathematics classroom, teachers must assign tasks and responsibilities to students in such a way that both boys and girls have active learning experiences with the technological tools employed.

2. Focus on Meaning

Focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning.

There is a long history of research, going back to the work of Brownell (1945,1947), on the effects of teaching for meaning and understanding. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on student learning, including better initial learning, greater retention and an increased likelihood that the ideas will be used in new situations.

Classroom Implications:

- Emphasize the mathematical meanings of ideas, including how the idea, concept or skill is connected in multiple ways to other mathematical ideas in a logically consistent and sensible manner.
- Create a classroom learning context in which students can construct meaning.
- Make explicit the connections between mathematics and other subjects.
- Attend to student meanings and student understandings.

3. Problem Solving

Students can learn both concepts and skills by solving problems.

Results from research suggest that students who develop conceptual understanding early perform best on procedural knowledge later. Students with good conceptual understanding are able to perform successfully on near-transfer tasks and to develop procedures and skills they have not been taught. Students with low levels of conceptual understanding need more practice in order to acquire procedural knowledge.

Classroom Implications:
There is evidence that students can learn new skills and concepts while they are working out solutions to problems. Development of more sophisticated mathematical skills can also be approached by treating their development as a problem for students to solve. Results from research suggest that it is not necessary for teachers to focus first on skill development and then move on to problem solving. Both can be done together. Skills can be developed on an as-needed basis, or their development can be supplemented through the use of technology. In fact, there is evidence that if students are initially drilled too much on isolated skills, they have a harder time making sense of them later.

4. Opportunities for Invention and Practice

Giving students both an opportunity to discover and invent new knowledge and an opportunity to practice what they have learned improves student achievement.

Data from the TIMSS video study show that over 90% of mathematics class time in the United States 8th-grade classrooms is spent practicing routine procedures, with the remaining time generally spent applying procedures in new situations. Virtually no time is spent inventing new procedures and analyzing unfamiliar situations. In contrast, students at the same grade level in typical Japanese classrooms spend approximately 40% of instructional time practicing routine procedures, 15% applying procedures in new situations, and 45% inventing new procedures and analyzing new situations.

Results from research suggest that students need opportunities for both practice and invention. Findings from a number of studies show that when students discover mathematical ideas and invent mathematical procedures, they have a stronger conceptual understanding of connections between mathematical ideas.

Classroom Implications:
Balance is needed between the time students spend practicing routine procedures and the time they devote to inventing and discovering new ideas. Teachers need not choose between these; indeed, they must not make a choice if students are to develop the mathematical power they need.

To increase opportunities for invention, teachers should frequently use non-routine problems, periodically introduce a lesson involving a new skill by posing it as a problem to be solved, and regularly allow students to build new knowledge based on their intuitive knowledge and informal procedures.

5. Openness to Student Solutions and Student Interactions

Teaching that incorporates students' intuitive solution methods can increase student learning, especially when combined with opportunities for student interaction and discussion.

Student achievement and understanding are significantly improved when teachers are aware of how students construct knowledge, are familiar with the intuitive solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in mathematics.

Structuring instruction around carefully chosen problems, allowing students to interact when solving problems, and then providing opportunities for them to share their solution methods result in increased achievement on problem-solving measures. These gains come without a loss of achievement in the skills and concepts measured on standardized achievement tests.

Classroom Implications:
Research results suggest that teachers should concentrate on providing opportunities for students to interact in problem-rich situations. Besides providing appropriate problem-rich situations, teachers must encourage students to find their own solution methods and give them opportunities to share and compare their solution methods and answers. One way to organize such instruction is to have students work in small groups initially and then share ideas and solutions in a whole-class discussion.

6. Small Group Learning

Using small groups of students to work on activities, problems and assignments can increase student mathematics achievement.

Davidson (1985) reviewed studies that compared student achievement in small group settings with traditional whole-class instruction. In more than 40% of these studies, students in the classes using small group approaches significantly outscored control students on measures of student performance. In only two of the 79 studies did control-group students perform better than the small group students, and in these studies there were some design irregularities. From a review of 99 studies of cooperative group-learning methods, Slavin (1990) concluded that cooperative methods were effective in improving student achievement. The most effective methods emphasized both group goals and individual accountability.

Classroom Implications:
When using small groups for mathematics instruction, teachers should:

- Choose tasks that deal with important mathematical concepts and ideas.
- Select tasks that are appropriate for group work.
- Consider having students initially work individually on a task and then follow with group work where students share and build on their individual ideas and work.
- Give clear instructions to the groups and set clear expectations for each (for each task or each group?).
- Emphasize both group goals and individual accountability.
- Choose tasks that students find interesting.
- Ensure that there is closure to the group work, where key ideas and methods are brought to the surface either by the teacher or the students, or both.

7. Whole-Class Discussion

Whole-class discussion following individual and group work improves student achievement.

Results from research suggest that whole class discussion can be effective when it is used for sharing and explaining the variety of solutions by which individual students

have solved problems. It allows students to see the many ways of examining a situation and the variety of appropriate and acceptable solutions. Wood (1999) found that whole-class discussion works best when discussion expectations are clearly understood. Students should be expected to evaluate each other's ideas and reasoning in ways that are not critical of the sharer. Students should be expected to be active listeners who participate in discussion and feel a sense of responsibility for each other's understanding.

Classroom Implications:
It is important that whole-class discussion follows student work on problem-solving activities. The discussion should be a summary of individual work in which key ideas are brought to the surface. This can be accomplished through students presenting and discussing their individual solution methods, or through other methods of achieving closure that are led by the teacher, the students, or both.

Whole-class discussion can also be an effective diagnosis tool for determining the depth of student understanding and identifying misconceptions. Teachers can identify areas of difficulty for particular students, as well as ascertain areas of student success or progress.

8. Focus on Number Sense

Teaching mathematics with a focus on number sense encourages students to become problem solvers in a wide variety of situations and to view mathematics as a discipline in which thinking is important.

Number sense relates to having an intuitive feel for number size and combinations, and the ability to work flexibly with numbers in problem situations in order to make sound decisions and reasonable judgments. It involves mentally computing, estimating, sensing number magnitudes, moving between representation systems for numbers, and judging the reasonableness of numerical results. Markovits and Sowder (1994) studied 7th-grade classes where special units on number magnitude, mental computation and computational estimation were taught. They determined that after this special instruction, students were more likely to use strategies that reflected sound number sense, and that this was a long-lasting change. In a study of second graders, Cobb (1991) and his colleagues found that students' number sense was improved by a problem-centered curriculum that emphasized student interaction and self-generated solution methods. Almost every student developed a variety of strategies to solve a wide range of problems. Students also demonstrated increased persistence in solving problems.

Classroom Implications:
Competence in the many aspects of number sense is an important mathematical outcome for students. Over 90% of the computation done outside the classroom is done without pencil and paper, using mental computation, estimation or a calculator.

However, in many classrooms, efforts to instill number sense are given insufficient attention.

As teachers develop strategies to teach number sense, they should strongly consider moving beyond a unit-skills approach (i.e. a focus on single skills in isolation) to a more integrated approach that encourages the development of number sense in all classroom activities, from the development of computational procedures to mathematical problem-solving.

9. Use Concrete Materials

Long-term use of concrete materials is positively related to increases in student mathematics achievement and improved attitudes towards mathematics.

In a review of activity-based learning in mathematics in kindergarten through grade 8, Suydam and Higgins (1977) concluded that using manipulative materials produces greater achievement gains than not using them. In a more recent meta-analysis of sixty studies (kindergarten through postsecondary) that compared the effects of using concrete materials with the effects of more abstract instruction, Sowell (1989) found that the long-term use of concrete materials by teachers knowledgeable in their use improved student achievement and attitudes.

Classroom Implications:
Results from research suggest that teachers use manipulative materials regularly in order to give students hands-on experience that helps them construct useful meanings for the mathematical ideas they are learning. Use of the same materials to teach multiple ideas over the course of schooling shortens the amount of time it takes to introduce the material and helps students see connections between ideas.

The use of concrete material should not be limited to demonstrations. It is essential that children use materials in meaningful ways rather than in a rigid and prescribed way that focuses on remembering rather than on thinking.

10. Use Calculators

Using calculators in the learning of mathematics can result in increased achievement and improved student attitudes.

Studies have consistently shown that thoughtful use of calculators improves student mathematics achievement and attitudes toward mathematics. From a meta-analysis of 79 non-graphing calculator studies, Hembree and Dessart (1986) concluded that use of hand-held calculators improved student learning. Analysis also showed that students

using calculators tended to have better attitudes towards mathematics and better self-concepts in mathematics than their counterparts who did not use calculators. They also found that there was no loss in student ability to perform paper-and-pencil computational skills when calculators were used as part of mathematics instruction.

Research on the use of graphing calculators has also shown positive effects on student achievement. Most studies have found positive effects on students' graphing ability, conceptual understanding of graphs and their ability to relate graphical representations to other representations. Most studies of graphing calculators have found no negative effect on basic skills, factual knowledge, or computational skills.

Classroom Implications:
One valuable use for calculators is as a tool for exploration and discovery in problem-solving situations and when introducing new mathematical content. By reducing computation time and providing immediate feedback, calculators help students focus on understanding their work and justifying their methods and results. The graphing calculator is particularly useful in helping to illustrate and develop graphical concepts and in making connections between algebraic and geometric ideas.

In order to accurately reflect their meaningful mathematics performance, students should be allowed to use their calculators in achievement tests. Not to do so is a major disruption in many students' usual way of doing mathematics, and an unrealistic restriction because when they are away from the school setting, they will certainly use a calculator in their daily lives and in the workplace.

Nine Essential Instructional Strategies

Researchers at Mid-continent Research for Education and Learning (McREL) have identified nine instructional strategies that are most likely to improve student achievement across all content areas and across all grade levels. These strategies are explained in the book Classroom Instruction That Works by Marzano, Pickering, and Pollock (2001).

1. Identifying similarities and differences
2. Summarizing and note taking
3. Reinforcing effort and providing recognition
4. Homework and practice
5. Nonlinguistic representations
6. Cooperative learning
7. Setting objectives and providing feedback
8. Generating and testing hypotheses
9. Cues, questions, and advance organizers

The following is an overview of the research behind these strategies as well as some practical applications for the classroom.

1. Identifying Similarities and Differences

The ability to break a concept into its similar and dissimilar characteristics allows students to understand (and often solve) complex problems by analyzing them in a more simple way. Teachers can either directly present similarities and differences, accompanied by deep discussion and inquiry, or simply ask students to identify similarities and differences on their own. While teacher-directed activities focus on identifying specific items, student-directed activities encourage variation and broaden understanding. Results from research also suggest that graphic forms are a good way to represent similarities and differences.

Applications:

- Use Venn diagrams or charts to compare and classify items.
- Engage students in comparing, classifying, and creating metaphors and analogies.

2. Summarizing and Note Taking

These skills promote greater comprehension by asking students to analyze a subject to expose what's essential and then put it in their own words. According to researchers, this requires substituting, deleting, and keeping some things and having an awareness of the basic structure of the information presented.

Applications:

- Provide a set of rules for creating a summary.
- When summarizing, ask students to question what is unclear, clarify those questions, and then predict what will happen next in the text.

Research shows that taking more notes is better than fewer notes, though verbatim note taking is ineffective because it does not allow time to process the information. Teachers should encourage and give time for review and revision of notes; notes can be the best study guides for tests.

Applications:

- Use teacher-prepared notes.
- Stick to a consistent format for notes, although students can refine the notes as necessary.

3. Reinforcing Effort and Providing Recognition

Effort and recognition speak to the attitudes and beliefs of students, and teachers must show the connection between effort and achievement. Although not all students realize the importance of effort, they can learn to change their beliefs to emphasize effort.

Applications:

- Share stories about people who succeeded by not giving up.
- Have students keep a log of their weekly efforts and achievements, reflect on it periodically, and even mathematically analyze the data.

Recognition was found to be most effective if it is contingent on the achievement of a certain standard. Also, symbolic recognition works better than tangible rewards.

Applications:

- Find ways to personalize recognition. Give awards for individual accomplishments.
- "Pause, Prompt, Praise." If a student is struggling, pause to discuss the problem, then prompt with specific suggestions to help her improve. If the student's performance improves as a result, offer praise.

4. Homework and Practice

Homework provides students with the opportunity to extend their learning outside the classroom. The amount of homework assigned should vary by grade level and that parent involvement should be minimal. Teachers should explain the purpose of homework to both the student and the parent or guardian, and teachers should try to give feedback on all homework assigned.

Applications:

- Establish a homework policy with advice-such as keeping a consistent schedule, setting, and time limit-that parents and students may not have considered.
- Tell students if homework is for practice or preparation for upcoming units.
- Maximize the effectiveness of feedback by varying the way it is delivered.

Students should adapt skills while they're learning them. Speed and accuracy are key indicators of the effectiveness of practice.

Applications:

- Assign timed quizzes for homework and have students report on their speed and accuracy.
- Focus practice on difficult concepts and set aside time to accommodate practice periods.

5. Nonlinguistic Representations

Knowledge is stored in two forms: linguistic and visual. The more students use both forms in the classroom, the more opportunity they have to achieve. Recently, use of

nonlinguistic representation has proven to not only stimulate but also increase brain activity.

Applications:

- Incorporate words and images using symbols to represent relationships.
- Use physical models and physical movement to represent information.

6. Cooperative Learning

Organizing students into cooperative groups yields a positive effect on overall learning. When applying cooperative learning strategies, keep groups small and don't overuse this strategy-be systematic and consistent in your approach.

Applications:

- When grouping students, consider a variety of criteria, such as common experiences or interests.
- Vary group sizes and objectives.
- Design group work around the core components of cooperative learning-positive interdependence, group processing, appropriate use of social skills, face-to-face interaction, and individual and group accountability.

7. Setting Objectives and Providing Feedback

Setting objectives can provide students with a direction for their learning. Goals should not be too specific; they should be easily adaptable to students' own objectives.

Applications:

- Set a core goal for a unit, and then encourage students to personalize that goal by identifying areas of interest to them. Questions like "I want to know" and "I want to know more about . . ." get students thinking about their interests and actively involved in the goal-setting process.
- Use contracts to outline the specific goals that students must attain and the grade they will receive if they meet those goals.

According to results from studies, feedback generally produces positive results. Teachers can never give too much; however, they should manage the form that feedback takes.

Applications:

- Make sure feedback is corrective in nature; tell students how they did in relation to specific levels of knowledge. Rubrics are a great way to do this.
- Keep feedback timely and specific.

- Encourage students to lead feedback sessions.

8. Generating and Testing Hypotheses

A deductive approach (using a general rule to make a prediction) to this strategy works best. Whether a hypothesis is induced or deduced, students should clearly explain their hypotheses and conclusions.

Applications:

- Ask students to predict what would happen if an aspect of a familiar system, such as the government or transportation, were changed.
- Ask students to build something using limited resources. This task generates questions and hypotheses about what may or may not work.

9. Cues, Questions, and Advance Organizers

Cues, questions, and advance organizers help students use what they already know about a topic to enhance further learning. These tools should be highly analytical, should focus on what is important, and are most effective when presented before a learning experience.

Applications:

- Pause briefly after asking a question. Doing so will increase the depth of your students' answers.
- Vary the style of advance organizer used: Tell a story, skim a text, or create a graphic image. There are many ways to expose students to information before they "learn" it.

Recommendations

To offer the classes needed to make Homer Community School competitive in mathematics with other schools in classes offered as well as test scores, we need to focus on the accelerated students as well as the students that need intervention. We have always had the classes and staff needed to focus on the average students. An additional staff member will allow us to offer "college-tract" classes to students at a younger age, eventually allowing us to offer more advanced math classes. At the same time, we could also assist students that need assistance to become more proficient in math.

Recommended Elementary Program Components

- Students will engage in 60 minutes of mathematics instruction or practice daily.
- Classroom teachers are to differentiate instruction to provide the appropriate instructional support to meet students' unique learning needs of all students.
- Intervention and Enrichment (I & E) Time is to be used for both reading and mathematics instruction.
- Students in first through sixth grades are to utilize FASTTMATH for 15 minutes three times a week.
- A Mathematics Interventionist is needed to assist in meeting the instructional needs of students with different abilities. (i.e. talented /gifted and lower ability students)

Recommended Class Additions for 7-12 Mathematics

- Jr. High Math – 7th Grade – 5th Hour
- Algebra 1 – 8th Grade – 2nd Hour, 7th Hour
- Geometry – 9th Grade – 3rd Hour
- Advanced Math – 11th Grade – 3rd Hour, 8th Hour
- Consumer Math – 1st Hour
- Personal Finance / Life Skills Math

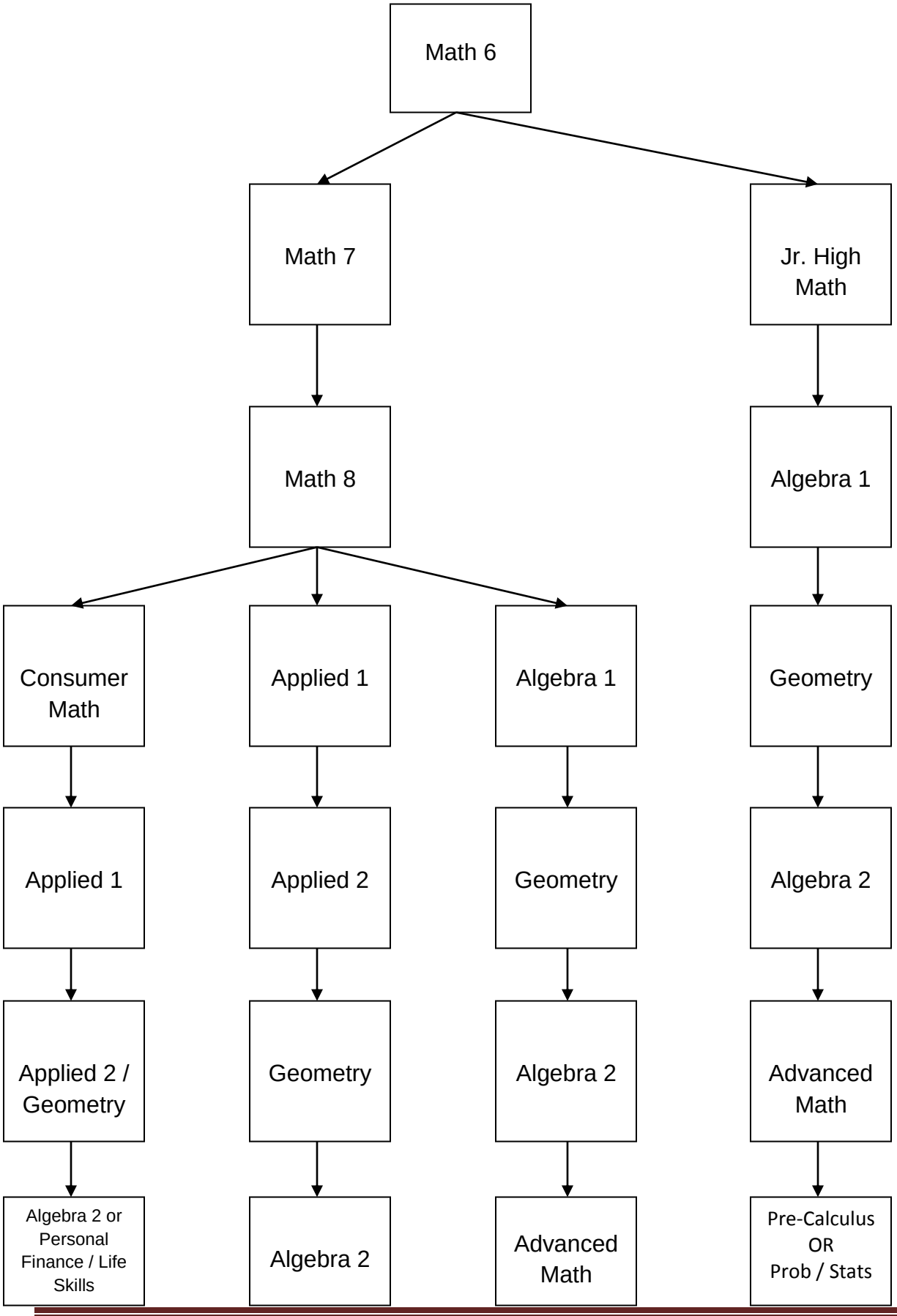
Additional Teacher Schedule

- *1st Hour – Consumer Math
- *2nd Hour – Algebra 1 (8th Grade)
- 3rd Hour – Math Intervention
- *4th Hour – Plan
- *5th Hour – Jr. High Math (7th Grade)
- 6th Hour – Math Intervention
- 7th Hour – Applied Math 3

- 8th Hour – Personal Finance / Life Skills Math

(Eventually, this teacher would also have to teach a Pre-Calculus or Probability / Statistics class.)

- * = class offered next year
- Would be nice to offer 8th Hour next year, but will take some time to develop a curriculum for the class.


Math 6
Math 7
Jr. High Math
Math 8
Algebra 1
Consumer Math
Applied 1
Algebra 1
Geometry
Applied 1
Applied 2
Geometry
Algebra 2
Applied 2 / Geometry
Geometry
Algebra 2
Advanced Math
Algebra 2 or Personal Finance / Life Skills
Algebra 2
Advanced Math
Pre-Calculus OR Prob / Stats

Curriculum Frameworks

Kindergarten

MA 0.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 0.1.1	Number System: Students will demonstrate, represent, and show relationships among whole numbers within the base-ten number system.
	MA 0.1.1.a Count , read and write numbers 0 – 20
	Count orally to 100
	Count by 10s to 100
	Count backwards form 10-0
	Count forward from a given number to 20
	MOVE TO K – count to 100
	MA 0.1.1.b Count objects using one-to-one correspondence 0 – 20
	MA 0.1.1.c Sequence objects using ordinal numbers (first through fifth)
	MA 0.1.1.d Match numerals to the quantities they represent 0 – 20, using a variety of models and representations
	MA 0.1.1.e Demonstrate and identify multiple equivalent representations for numbers 1 – 10 (e.g., 10 is 1 and 9; 10 is 6 and 4)
	MA 0.1.1.f Demonstrate relative position of whole numbers 0 – 10 (e.g., 5 is between 2 and 10; 7 is greater than 3)
MA 0.1.2	Operations: Students will demonstrate the meaning of addition and subtraction with whole numbers
	MA 0.1.2.a Use objects and words to explain the meaning of addition as a joining action (e.g., Two girls are sitting at a table. Two more girls join them. How many girls are sitting at the table?)
	MA 0.1.2.b Use objects and words to explain the meaning of addition as parts of a whole (e.g., Three boys and two girls are going to the zoo. How many children are going to the zoo?)
	MA 0.1.2.c Use objects and words to explain the meaning of subtraction as a separation action (e.g., Five girls are sitting at a table. Two girls leave. How many girls are left sitting at the table?)
	MA 0.1.2.d Use objects and words to explain the meaning of subtraction as

	finding part of a whole (e.g., Jacob has 5 pencils. Three are blue and the rest are red. How many red pencils does Jacob have?)
	Use objects to represent addition and subtraction – sums/differences 0-10
MA 0.1.3	Computation: Mastery not expected at this level.
	Fluently add and subtract – with objects on facts to/from 10
	Fluently add and subtract – answer facts to/from 5 - AUTOMATIC
MA 0.1.4	Estimation: Mastery not expected at this level.
MA 0.2	**Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.**
MA 0.2.1	Characteristics: Students will identify two dimensional geometric shapes.
	MA 0.2.1.a Sort and name two-dimensional shapes (e.g., square, circle, rectangle, triangle)
	2-D and 3-D shapes: identify square, circle, rectangle, triangle, sphere, cube, cylinder, and cone
MA 0.2.2	Coordinate Geometry: Mastery not expected at this level.
MA 0.2.3	Transformations: Mastery not expected at this level.
MA 0.2.4	Spatial Modeling: Students will communicate relative positions in space.
	MA 0.2.4.a Demonstrate positional words (e.g., above/below, near/far, over/ under, in/out, down/up, around/through)
MA 0.2.5	Measurement: Students will measure using nonstandard units and time.
	MA 0.2.5.a Identify the name and amount of a penny, nickel, dime, and quarter
	MA 0.2.5.b Identify time to the hour
	MA 0.2.5.c Measure using nonstandard units

	MA 0.2.5.d Compare objects according to length
	Compare measurements – by length, weight, and attributes
MA 0.3	**Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.**
MA 0.3.1	Relationships: Students will sort, classify, and order objects by relationships.
	MA 0.3.1.a Sort by color, shape, or size
	MA 0.3.1.b Create own rule for sorting other than color, shape, and size
	Patterns – recognize, describe, extend, and analyze using shapes
MA 0.3.2	Modeling in Context: Students will use objects as models to represent mathematical situations.
	MA 0.3.2.a Model situations that involve the addition and subtraction of whole numbers 0 – 10 using objects
MA 0.3.3	Procedures: Students will use concrete and verbal representations to solve number stories. MA 0.3.3.a Use objects to solve addition and subtraction of whole numbers 0 – 10
MA 0.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 0.4.1	Display and Analysis: Students will sort, classify, represent, describe, and compare sets of objects.
	MA 0.4.1.a Sort and classify objects according to an attribute (e.g., size, color, shape)
	MA 0.4.1.b Identify the attributes of sorted data
	MA 0.4.1.c Compare the attributes of the data (e.g., most, least, same)
MA 0.4.2	Predictions and Inferences: Mastery not expected at this level.

MA 0.4.3	Probability: Mastery not expected at this level.

Kindergarten Mathematics Vocabulary

above *	difference	place value
add*	different*	plus*
addend	digit	quantity
after*	equal to*	rectangle*
array	equation	sequence
attribute	expression	shape*
before*	face	shorter*
behind*	greater than	side*
below*	heavier*	similar
beside	height	size*
between*	hexagon	sort*
by	in front of*	sphere
calendar *	length*	square*
category	less than*	subtract
circle*	lighter*	sum*
classify	line	taller*
compare*	longer*	tens
compose	next to*	3-dimensional
cone	number*	triangle*
count*	numeral	2-dimensional
cube	ones	vertex
cylinder	opposite*	weight*
day*	pair	
decompose	pattern*	

(* denotes mastery of vocabulary word)

Grade 1

MA 1.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 0.1.1	Number System: Students will demonstrate, represent, and show relationships among whole numbers within the base-ten number system.
	MA 1.1.1.a Count , read and write numbers 0 – 100
	Count forward from a given number 20-100
	MA 1.1.1.b Count by multiples of 2 up to 50
	MA 1.1.1.c Count by multiples of 5 up to 100
	MA 1.1.1.d Count by multiples of 10 up to 100
	MA 1.1.1.e Sequence objects using ordinal numbers (first through tenth)
	MA 1.1.1.f Count backwards from 10 – 0
	Count backwards from 20
	MA 1.1.1.g Connect number words to the quantities they represent 0 – 20
	MA 1.1.1.h Demonstrate and identify multiple equivalent representations for numbers 1 – 100 (e.g., 23 is 2 tens and 3 ones; 23 is 1 ten and 13 ones; 23 is 23 ones)
	MA 1.1.1.i Compare and order whole numbers 0 – 100
	MA 1.1.1.j Demonstrate relative position of whole numbers 0 – 100 (e.g., 52 is between 50 and 60; 83 is greater than 77)
	Fractions – understand the concept of equal parts and identify 1/2
	When given a number 1-100: mentally find 1 more or 1 less
	Identify odd and even numbers 1-10
MA 1.1.2	Operations: Students will demonstrate the meaning of addition and subtraction with whole numbers
	MA 1.1.2.a Use objects, drawings, words, and symbols to explain addition as a joining action
	MA 1.1.2.b Use objects, drawings, words, and symbols to explain addition as parts of a whole

	MA 1.1.2.c Use objects, drawings, words, and symbols to explain subtraction as a separation action
	MA 1.1.2.d Use drawings, words, and symbols to explain subtraction as finding part of a whole
	MA 1.1.2.e Use objects, drawings, words, and symbols to explain subtraction as a comparison (e.g., Nancy has 8 hair ribbons. Jane has 5 hair ribbons. How many more hair ribbons does Nancy have than Jane?
	Use objects to represent addition and subtraction (part/part/whole and joining/separation) facts to/from 20
	Compute addition and subtraction word problems – sums and differences to/from 20
MA 1.1.3	Computation: Students will compute fluently and accurately using appropriate strategies and tools.
	MA 1.1.3.a Fluently add whole number sums up to 10
	MA 1.1.3.b Fluently subtract whole number differences from 10
	MA 1.1.3.c Add and subtract two-digit numbers without regrouping
	Add and subtract 2 digit numbers to 100 without regrouping
	MA 1.1.3.d Use a variety of methods and tools to compute sums and differences (e.g., models, mental computation, paper-pencil)
	Commutative property – students demonstrate property and are able to use term.
	Variables (8+_=11 or 5+_-3) up to 5
MA 1.1.4	Estimation: Mastery not expected at this level.
MA 1.2 Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 1.2.1	Characteristics: Students will identify characteristics of two-dimensional geometric shapes.
	MA 1.2.1.a Compare two-dimensional shapes (e.g., square, circle, rectangle, triangle)

	MA 1.2.1.b Describe attributes of two-dimensional shapes (e.g., square, circle, rectangle, triangle)
	3-D shapes – identify pyramid and rectangular prism
MA 1.2.2	Coordinate Geometry: Students will identify locations on a number line.
	MA 1.2.2.a Identify the position of a whole number on a horizontal number line
MA 1.2.3	Transformations: Students will identify a line of symmetry.
	MA 1.2.3.a Identify one line of symmetry in two-dimensional shapes (e.g., circle, square, rectangle, triangle)
MA 1.2.4	Spatial Modeling: Students will communicate relative positions in space and create two-dimensional shapes.
MA 1.2.5	Measurement: Students will measure using standard units, time, and money.
	MA 1.2.5.a Count like coins to $1.00
	Money – using the 4 basic coins, add like coins to make a $1.00
	MA 1.2.5.b Identify time to the half hour
	Tell time to the hour and half hour using digital and analog clocks. Also use time to tell about past, present, and future events.
	MA 1.2.5.c Identify past, present, and future as orientation in time
	MA 1.2.5.d Select an appropriate tool for the attribute being measured e.g., clock, calendar, thermometer, scale, ruler)
	Identify and select appropriate tools to solve a problem – clock, calendar, thermometer, scale and ruler
	MA 1.2.5.e Measure length using inches
	MA 1.2.5.f Compare and order objects according to length
MA 1.3 Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within	

mathematics and across disciplines.	
MA 1.3.1	Relationships: Students will identify and describe relationships.
	MA 1.3.1.a Sort or order objects by their attributes (e.g., color, shape, size, number) then identify the classifying attribute
	MA 1.3.1.b Create multiple rules for sorting beyond color, shape, and size
	MA 1.3.1.c Identify, describe, and extend patterns (e.g., patterns with a repeating core)
	MA 1.3.1.d Use <, =, > to compare quantities
	Compare numbers to 100 using greater than/less than/equal to symbols
MA 1.3.2	Modeling in Context: Students will use objects and pictures as models to represent mathematical situations.
	MA 1.3.2.a Model situations that involve the addition and subtraction of whole numbers 0 – 20, using objects and pictures
	MA 1.3.2.b Describe and model quantitative change (e.g., a student growing taller)
MA 1.3.3	Procedures: Students will use concrete, verbal, and visual representations to solve number sentences.
	MA 1.3.3.a Write number sentences to represent fact families
	MA 1.3.3.b Use concrete, pictorial, and verbal representations of the commutative property of addition
MA 1.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 1.4.1	Display and Analysis: Students will sort, classify, organize, describe, and compare data.
	MA 1.4.1.a Sort and classify objects by more than one attribute
	MA 1.4.1.b Organize data by using concrete objects
	MA 1.4.1.c Represent data by using tally marks

	MA 1.4.1.d Compare and interpret information from displayed data (e.g., more, less, fewer)
	Organize, compare, and interpret data – up to 3 categories and find total number of data points
MA 1.4.2	Predictions and Inferences: Mastery not expected at this level.
MA 1.4.3	Probability: Mastery not expected at this level.

First Grade Mathematics Vocabulary

add	graph*	pattern
addend	greater	penny*
addition sentence*	greater than*	picture graph*
after	greatest	plane shape
afternoon*	half hour	plus
bar graph	half past	pound
before	halves	present*
between	hour*	quart
calendar	hour hand*	quarter*
cent*	hundreds	rectangle
centimeter	impossible	rectangular prism
circle	inch*	related facts
color	join	repeating pattern
compare	least*	ruler*
cone	less*	same shape
corner*	less likely	same size
count*	less than	side
count back	line of symmetry	size
count on	liter	shape
cube cylinder	measure*	slide
day	minus*	solid figure
difference	minute*	sort
digit	minute hand*	sphere
dime*	month	square
dollar	more likely	subtract*
double*	more than*	subtraction sentence
equal parts	morning	sum
equal to	nickel*	symmetry*
estimate	night	tally mark*
even	number	temperature*
face	number line*	tens*

fact family	o'clock	thermometer*
feet	odd	thirds
fewer than*	one fourth	triangle
flat surface	one half	turn
flip	one third	vertex
foot	ones*	week
fourth	past*	whole*
fraction*	pair*	year*
future		

(* denotes mastery of vocabulary word)

Grade 2

MA 2.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 2.1.1	Number System: Students will demonstrate, represent, and show relationships among whole numbers within the base-ten number system.
	MA 2.1.1.a Read and write numbers 0 – 1,000 (e.g., count numbers from 400 – 500; write numbers from 400 – 500)
	Read and write numbers 10-1000
	Count by 2s from 50-100
	MA 2.1.1.b Count by multiples of 2 up to 100
	MA 2.1.1.c Count backwards from 20 – 0
	MA 2.1.1.d Connect number words to the quantities they represent 0 – 100
	MA 2.1.1.e Demonstrate multiple equivalent representations for numbers 1 – 1,000 (e.g., 423 is 4 hundreds, 2 tens and 3 ones; 423 is 3 hundreds 12 tens and 3 ones)
	MA 2.1.1.f Compare and order whole numbers 0 – 1,000
	Compare numbers to 1000 using greater than/less than/equal to symbols
	MA 2.1.1.g Demonstrate relative position of whole numbers 0 – 1,000 (e.g., 624 is between 600 and 700; 593 is greater than 539)
	MA 2.1.1.h Use visual models to represent fractions of one-half as a part of a whole
	Fractions – 1/3 and 1/4
	Count by 100s from 0-1000
	Identify odd and even numbers 0-20
	When given a number 1-100: mentally find 10 more or 10 less

MA 2.1.2	Operations: Students will demonstrate the meaning of addition and subtraction with whole numbers.
	MA 2.1.2.a Use objects, drawings, words, and symbols to explain the relationship between addition and subtraction (e.g., if 2 + 3 = 5 then 5 – 3 = 2)
	MA 2.1.2.b Use objects, drawings, words, and symbols to explain the use of subtraction to find a missing addend (e.g., if 3 + __ = 7, then 7- 3 = __.)
MA 2.1.3	Computation: Students will compute fluently and accurately using appropriate strategies and tools.
	MA 2.1.3.a Fluently add whole number facts with sums to 20
	MA 2.1.3.b Fluently subtract whole number facts with differences from 20
	MA 2.1.3.c Add and subtract three-digit whole numbers with regrouping
	MA 2.1.3.d Use a variety of methods and tools to compute sums and differences (e.g., models, mental computation, paper–pencil)
	Add and subtract 2 digit numbers to 100 with regrouping
	Variables (8+_=11 or 5=_-3) up to 10
MA 2.1.4	Estimation: Students will estimate and check reasonableness of answers using appropriate strategies and tools.
	MA 2.1.4.a Estimate the results of two-digit whole number sums and differences and check the reasonableness of such results
	MA 2.1.4 b Estimate the number of objects in a group
MA 2.2 Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 2.2.1	Characteristics: Students will describe characteristics of two-dimensional shapes and identify three-dimensional objects.
	MA 2.2.1.a Describe attributes of two-dimensional shapes (e.g., trapezoid, parallelogram)
	MA 2.2.1.b Determine if two shapes are congruent

	MA 2.2.1.c Compare two-dimensional shapes (e.g., trapezoid, parallelogram)
	MA 2.2.1.d Identify solid shapes (e.g., triangular prism, rectangular prisms, cones, cylinders, pyramids, spheres)
	3D shapes – identify triangular prism
	3D shapes attributes – identify vertices, sides/faces, and edges
MA 2.2.2	Coordinate Geometry: Students will describe direction on a positive number line.
	MA 2.2.2.a Identify numbers using location on a vertical number line
	MA 2.2.2.b Compare whole numbers using location on a horizontal number line
	MA 2.2.2.c Identify the direction moved for adding and subtracting using a horizontal number line
MA 2.2.3	Transformations: Students will identify lines of symmetry.
	MA 2.2.3.a Identify lines of symmetry in two-dimensional shapes
	MA 2.2.3.b Draw a line of symmetry in two-dimensional shapes
MA 2.2.4	Spatial Modeling: Students will create two-dimensional shapes.
	MA 2.2.4.a Sketch two-dimensional shapes (e.g., trapezoid, parallelogram)
MA 2.2.5	Measurement: Students will measure using standard units, time and money.
	MA 2.2.5.a Count mixed coins to $1.00
	Money – use mixed coins to count to $1.00. Also use $ and cent sign accurately
	MA 2.2.5.b Identify time to 5 minute intervals
	Tell time on analog and digital clocks to 5 min. intervals. Also able to distinguish AM and PM
	MA 2.2.5.c Identify and use appropriate tools for the attribute being measured (e.g., clock, calendar, thermometer, scale, ruler)

	Use appropriate tools to solve a problem – clock, calendar, thermometer, scale, and ruler
	MA 2.2.5.d Measure length using feet and yards
	MA 2.2.5.e Compare and order objects using inches, feet and yards
MA 2.3 Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 2.3.1	Relationships: Students will identify, describe, and extend relationships.
	MA 2.3.1.a Create and describe patterns using concrete and pictorial representations
	Associative property – students demonstrate property and are able to use vocal word.
MA 2.3.2	Modeling in Context: Students will use objects, pictures, and symbols as models to represent mathematical situations.
	MA 2.3.2.a Model situations that involve the addition and subtraction of whole numbers 0 – 100, using objects and number lines
	MA 2.3.2.b Describe and model quantitative change involving addition (e.g., a student grew 2 inches)
MA 2.3.3	Procedures: Students will use concrete, verbal, visual, and symbolic representations to solve number sentences.
	MA 2.3.3.a Use symbolic representations of the commutative property of addition (e.g., 2 + 3 = Δ + 2)
MA 2.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 2.4.1	Display and Analysis: Students will organize, display, compare, and interpret data.
	MA 2.4.1.a Represent data using pictographs
	Graphs – use pictographs, bar graphs, and line plot to represent and interpret up to 4 categories

	MA 2.4.1.b Interpret data using pictographs (e.g., 7 more; 2 less; 12 all together)
MA 2.4.2	Predictions and Inferences: Mastery not expected at this level.
MA 2.4.3	Probability: Mastery not expected at this level.

Second Grade Mathematics Vocabulary

add	equation	one-third*
addend	estimate*	ones
a.m.	even number*	penny
analog clock*	expanded form	pentagon
angle*	expression	picture graph
array	face	place value*
Associative Property of Addition	fact families*	p.m.
addition	foot*	pyramid*
attribute	fourths	quadrilateral
bar graph*	fraction	quarter
base 10 blocks*	geometric solid	quarter of
category	greater than	quarter-hour
cent	half circle	rectangle
centimeter*	half dollar*	rectangular prism*
circle	half hour	regroup*
classify	halves	second*
clock face	hexagon	sequence
closed figure	hour	side of a shape
Commutative Property of Addition	hour hand	similar*
compare*	hundreds*	solid figure*
compose	inch	sort
cone*	key	sphere*
count back	length	square
count on	less than	standard form
counting up	line*	subtract
cube*	line plot	sum
customary system	meter	tens
cylinder*	metric system	thirds
data*	minute	three-digit number*
decompose	minute hand	three-dimensional*
difference	money	time
digit*	month*	triangle
digital clock*	nickel	two-dimensional*
dime	number	unit

dollar*	number line	vertex (vertices)
doubles	numeral	week*
edge	odd number*	weight
equal	one-fourth*	whole numbers
equal groups	one-half*	word form
equal shares	one hundred	

(* denotes mastery of vocabulary word)

Grade 3

MA 3.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 3.1.1	Number System: Students will represent and show relationships among positive rational numbers within the base-ten number system.
	MA 3.1.1.a Read and write numbers to one-hundred thousand (e.g., 4,623 is the same as four thousand six hundred twenty three)
	MA 3.1.1.b Count by multiples of 5 to 200
	MA 3.1.1c Count by multiples of 10 to 400
	MA 3.1.1.d Count by multiples of 100 to 1,000
*	MA 3.1.1.e Demonstrate multiple equivalent representations for numbers up to 10,000 (e.g., 10 tens is 1 hundred; 10 ten thousands is 1 hundred thousand; 2,350 is 235 tens; 2,350 is 2,000 + 300 + 50; 2,350 is 23 hundreds and 5 tens)
	MA 3.1.1.f Demonstrate multiple equivalent representations for decimal numbers through the tenths place (e.g., 3 and 6 tenths is 3.6; 7.4 is 7 + .4)
*	MA 3.1.1.g Compare and order whole numbers through the thousands
*	MA 3.1.1.h Find parts of whole and parts of a set for ½, ⅓, or ¼
*	Plot fractions on a number line
*	Recognize and find equivalent fractions
*	MA 3.1.1.i Round a given number to tens, hundreds, or thousand
MA 3.1.2	Operations: Students demonstrate the meaning of multiplication with whole numbers.
*	MA 3.1.2.a Represent multiplication as repeated addition using objects, drawings, words, and symbols (e.g., 3 x 4 = 4 + 4 + 4)
	MA 3.1.2.b Use objects, drawings, words and symbols to explain the relationship between multiplication and division (e.g., if 3 x 4 = 12 then 12 ÷

	3 = 4.)
	MA 3.1.2.c Use drawings, words, and symbols to explain the meaning of the factors and product in a multiplication sentence (e.g., in 3 x 4 = 12, 3 and 4 are factors and 12 is the total or product. The first factor (3) tells how many sets while the second factor tells how many are in each set. Another way to say this is that 3 groups of 4 equals 12 total.)
*	MA 3.1.2.d Use drawings, words, and symbols to explain the meaning of multiplication using an array (e.g., an array with 3 rows and 4 columns represents the multiplication sentence 3 x 4 = 12)
MA 3.1.3	Computation: Students will compute fluently and accurately using appropriate strategies and tools.
	MA 3.1.3.a Compute whole number multiplication facts 0 – 10 fluently
	MA 3.1.3.b Add and subtract through four-digit whole numbers with regrouping
	MA 3.1.3.c Select and apply the appropriate methods of computation when problem solving with four-digit whole numbers through the thousands (e.g., models, mental computation, paper-pencil)
MA 3.1.4	Estimation: Students will estimate and check reasonableness of answers using appropriate strategies and tools.
*	MA 3.1.4.a Estimate the two-digit product of whole number multiplication and check the reasonableness
MA 3.2 Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 3.2.1	Characteristics: Students will identify characteristics and describe properties of two-dimensional shapes and three-dimensional objects.
*	MA 3.2.1.a Identify the number of sides, angles, and vertices of two-dimensional shapes
*	MA 3.2.1.b Identify congruent two-dimensional figures given multiple two-dimensional shapes
	MA 3.2.1.c Identify lines, line segments, rays, and angles

	MA 3.2.1.d Describe attributes of solid shapes (e.g., triangular prism, rectangular prisms, cones, cylinders, pyramids, spheres)
MA 3.2.2	Coordinate Geometry: Students will identify distances on a number line.
	MA 3.2.2.a Draw a number line and plot points
	Use a number line to compare equivalent fractions to 1/2
	Compare and order fractions
*	MA 3.2.2.b Determine the distance between two whole number points on a number line
MA 3.2.3	Transformations: Students will draw all lines of symmetry.
	MA 3.2.3.a Draw all possible lines of symmetry in two-dimensional shapes
MA 3.2.4	Spatial Modeling: Students will create two-dimensional shapes and three-dimensional objects.
	MA 3.2.4.a Sketch and label lines, rays, line segments, and angles
	MA 3.2.4.b Build three-dimensional objects (e.g., using clay for rectangular prisms, cone, cylinder)
MA 3.2.5	Measurement: Students will apply appropriate procedures and tools to determine measurements using customary and metric units.
	MA 3.2.5.a Select and use appropriate tools to measure perimeter of simple two-dimensional shapes (e.g., triangle, square, rectangle)
	MA 3.2.5.b Count mixed coins and bills greater than $1.00
	MA 3.2.5.c Identify time of day (e.g., am, pm, noon, midnight)
	MA 3.2.5.d State multiple ways for the same time using 15 minute intervals (e.g., 2:15, or quarter past 2, 2:45 or a quarter until 3)
	Telling time to the minute
*	MA 3.2.5.e Identify the appropriate customary unit for measuring length, weight, and capacity/volume
	MA 3.2.5.f Measure length to the nearest ½ inch and centimeter (e.g.,

	requires rounding)
*	MA 3.2.5.g Compare and order objects according to length using centimeters and meters
	Measure length to the nearest ¼ inch
MA 3.3 Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 3.3.1	Relationships: Students will represent relationships.
*	MA 3.3.1.a Identify, describe, and extend numeric and non-numeric patterns
	MA 3.3.1.b Identify patterns using words, tables, and graphs
MA 3.3.2	Modeling in Context: Students will create and use models to represent mathematical situations.
*	MA 3.3.2.a Model situations that involve the addition and subtraction of whole numbers using objects, number lines, and symbols
	MA 3.3.2.b Describe and model quantitative change involving subtraction (e.g., temperature dropped two degrees)
MA 3.3.3	Procedures: Students will identify and apply properties of whole numbers to solve equations involving addition and subtraction.
	MA 3.3.3.a Use symbolic representation of the identity property of addition (e.g., 3 = 0 + 3)
*	MA 3.3.3.b Solve simple one-step whole number equations involving addition and subtraction (e.g., $\Delta + 2 = 3$)
	MA 3.3.3.c Explain the procedure(s) used in solving simple one-step whole number equations involving addition and subtraction
MA 3.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 3.4.1	Display and Analysis: Students will organize, display, compare, and

	interpret data.
*	MA 3.4.1.a Represent data using horizontal and vertical bar graphs
	MA 3.4.1.b Use comparative language to describe the data (e.g., increasing, decreasing)
*	MA 3.4.1.c Interpret data using horizontal and vertical bar graphs
MA 3.4.2	Predictions and Inferences: Mastery not expected at this level.
MA 3.4.3	Probability: Students will find and describe experimental probability.
	MA 3.4.3.a Perform simple experiments (e.g., flip a coin, toss a number cube, spin a spinner) and describe outcomes as possible, impossible, or certain

Third Grade Mathematics Vocabulary

acute angle*	fraction	pound*
addend*	gallon*	product*
area*	intersecting lines*	pyramid
array*	line graph*	quadrilateral*
Associative property*	line plot*	quantity*
bar graph	line segment*	quotient
Commutative Property*	meter*	range
cone	mile*	ray*
congruent*	mode	rectangular prism
cube	multiply*	right angle*
cylinder	numeral*	rounding*
decimal point*	numerator	sequence
denominator	obtuse angle*	sphere
difference*	ordinal number*	sum
divide*	parallel lines*	symmetry
dividend	perimeter*	tally mark
division	pictograph*	vertex*
divisor	place value	volume*
equation*	point*	weight*

fact family	polygon*	yard*
factor*		

(* denotes mastery of vocabulary word)

Grade 4

MA 4.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 4.1.1	Number System: Students will represent and show relationships among positive rational numbers within the base-ten number system.
	MA 4.1.1.a Read and write numbers through the millions (e.g., 2,347,589 is the same as 2 million three hundred forty seven thousand five hundred eighty nine)
*	MA 4.1.1.b Demonstrate multiple equivalent representations for decimal numbers through the hundredths place (e.g., 2 and 5 hundredths is 2.05; 6.23 is 6 + .2 +.03)
*	MA 4.1.1.c Compare and order whole numbers and decimals through the hundredths place (e.g., money)
	MA 4.1.1.d Classify a number as even or odd
	Factor pairs, multiples, prime and composite numbers
*	MA 4.1.1.e Represent a fraction as parts of a whole and/or parts of a set
	Write fraction as a decimal or other way
*	MA 4.1.1.f Use visual models to find equivalent fractions (e.g., $\frac{2}{4}=\frac{1}{2}, \frac{2}{8}=\frac{1}{4}, 1=\frac{2}{2}=\frac{5}{5}, \frac{3}{3}$
	Find common denominators
	MA 4.1.1.g Determine the size of a fraction relative to one half using equivalent forms (e.g., Is 3/8 more or less than one half?)
	Multiply a fraction by a whole number
	Find equivalent fractions in multiples of 10
*	MA 4.1.1.h Locate fractions on a number line

	Add and subtract fractions
	Add and subtract mixed numbers
	MA 4.1.1.i Round a whole number to millions
MA 4.1.2	Operations: Students will demonstrate the meaning of division with whole numbers.
*	MA 4.1.2.a Use drawings, words, and symbols to explain the meaning of division [(e.g., as repeated subtraction: Sarah has 24 candies. She put them into bags of 6 candies each. How many bags did Sarah use?) (e.g., as equal sharing: Paul has 24 candies. He wants to share them equally among his 6 friends. How many candies will each friend receive?)]
	Solve word problems with fractions
MA 4.1.3	Computation: Students will compute fluently and accurately using appropriate strategies and tools.
	MA 4.1.3.a Compute whole number division facts 0 – 10 fluently
*	MA 4.1.3.b Add and subtract decimals to the hundredths place (e.g., money)
*	MA 4.1.3.c Multiply two-digit whole numbers
	MA 4.1.3.d Divide a three-digit number with one digit divisor with and without a remainder
*	MA 4.1.3.e Mentally compute multiplication and division involving powers of 10
*	MA 4.1.3.f Select and apply the appropriate method of computation when problem solving (e.g., models, mental computation, paper-pencil)
MA 4.1.4	Estimation: Students will estimate and check reasonableness of answers using appropriate strategies and tools.
	MA 4.1.4.a Estimate the three-digit product and the two-digit quotient of whole number multiplication and division and check the

	reasonableness
MA 4.2 Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 4.2.1	Characteristics: Students will classify two-dimensional shapes and three-dimensional objects.
*	MA 4.2.1.a Identify two- and three-dimensional shapes according to their sides and angle properties
	Identify two dimensional shapes by sides and angles
	Add and subtract angle degrees
	Use the four basic operations to solve word problems
*	MA 4.2.1.b Classify an angle as acute, obtuse, and right
	Use a protractor to measure degrees
*	MA 4.2.1.c Identify parallel, perpendicular, and intersecting lines
	MA 4.2.1.d Identify the property of congruency when dealing with plane geometric shapes
MA 4.2.2	Coordinate Geometry: Students will describe locations using coordinate geometry.
*	MA 4.2.2.a Identify the ordered pair of a plotted point in first quadrant by its location (e.g., (2, 3) is a point two right and three up from the origin)
MA 4.2.3	Transformations: Students will identify simple transformations.
	MA 4.2.3.a Given two congruent geometric shapes, identify the transformation (e.g., translation, rotation, reflection) applied to an original shape to create a transformed shape
MA 4.2.4	Spatial Modeling: Student will use geometric models to solve problems.

	MA 4.2.4.a Given a geometric model, use it to solve a problem (e.g., what shapes make a cylinder; streets run parallel and perpendicular)
MA 4.2.5	Measurement: Students will apply appropriate procedures and tools to estimate and determine measurement using customary and metric units.
	MA 4.2.5.a Select and use appropriate tools to measure perimeter of polygons
	Use appropriate tool to measure polygon perimeter
*	MA 4.2.5.b Identify time to the minute on an analog clock
*	MA 4.2.5.c Solve problems involving elapsed time
*	MA 4.2.5.d Identify the appropriate metric unit for measuring length, weight, and capacity/volume (e.g., cm, m, Km; g, Kg; mL, L)
	Compare units of measurement
	Solve word problems using customary units of volume and capacity
	Determine area and unit of a square and rectangle
	MA 4.2.5.e Estimate and measure length using customary (nearest ½ inch) and metric (nearest centimeter) units
	Measure length to the nearest ¼ inch
	MA 4.2.5.f Measure weight and temperature using customary units
*	MA 4.2.5.g Compute simple unit conversions for length within a system of measurement
MA 4.3 Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 4.3.1	Relationships: Students will represent and analyze relationships.
	MA 4.3.1.a Describe, extend, and apply rules about numeric patterns

	MA 4.3.1.b Represent and analyze a variety of patterns using words, tables, and graphs
*	MA 4.3.1.c Use ≥, ≤ symbols to compare quantities
*	MA 4.3.1.d Select appropriate operational and relational symbols to make a number sentence true
MA 4.3.2	Modeling in Context: Students will create and use models to represent mathematical situations.
	MA 4.3.2.a Model situations that involve the multiplication of whole numbers using number lines and symbols
	Multiplicative comparisons
	MA 4.3.2.b Describe and model quantitative change involving multiplication (e.g., money doubling)
	Multiplicative comparisons with word problems
MA 4.3.3	Procedures: Students will identify and apply properties of whole numbers to solve equations involving multiplication and division.
	MA 4.3.3.a Represent the idea of a variable as an unknown quantity using a letter or a symbol (e.g., n + 3, b – 2)
	MA 4.3.3.b Use symbolic representation of the identity property of multiplication (e.g., 5 * 1 = 5)
*	MA 4.3.3.c Use symbolic representations of the commutative property of multiplication (e.g., 2 * 3 = Δ * 2)
*	MA 4.3.3.d Solve simple one-step whole number equations (e.g., x + 2 = 3, 3 * y = 6)
	Solve multi-step problems using the four opeations
	MA 4.3.3.e Explain the procedure(s) used in solving simple one-step whole number equations
MA 4.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections	

within mathematics and across disciplines.	
MA 4.4.1	Display and Analysis: Students will organize, display, compare, and interpret data.
	Collect data using observations and surveys
	MA 4.4.1.a Represent data using dot/line plots
*	MA 4.4.1.b Compare different representations of the same data
	Describe and compare data sets
*	MA 4.4.1.c Interpret data and draw conclusions using dot/line plots
	MA 4.4.1.d Find the mode and range for a set of whole numbers
	MA 4.4.1.e Find the whole number mean for a set of whole numbers
MA 4.4.2	Predictions and Inferences: Students will construct predictions based on data.
*	MA 4.4.2.a Make predictions based on data to answer questions from tables and bar graphs
MA 4.4.3	Probability: Students will find, describe, and compare experimental probabilities.
	MA 4.4.3.a Perform simple experiments and compare the degree of likelihood (e.g., more likely, equally likely, or less likely)

Fourth Grade Mathematics Vocabulary

acute angle	evaluate	number line
add	expanded form*	numerator*
addend	expression*	obtuse angle
additive comparison	fact family	order of operations
algorithm*	factor	ounce
angle	factor pairs	parallel lines

angle measure	foot	parentheses*
arc	formula	pattern
area	fraction	perimeter
area model	function table	period
array	gallon	perpendicular lines*
associative property of addition	gram	pint*
associative property of multiplication	greater than	place value
attribute*	hour	plane figure*
benchmark fractions*	hundredth*	point
capacity*	hundredths	pound
centimeter	identity property of addition*	prime number
circle	identity property of multiplication*	protractor
classify*	improper fraction*	quart
common denominator	inch	quotient*
commutative property of addition	increase*	range*
commutative property of multiplication	intersecting lines	ray
compare	inverse operations	reasonableness
comparison bars	kilogram	relate facts
compose	kilometer	remainder*
composite number*	length	right angle
congruent	less than	right triangle
cup*	like denominators	round a whole number
customary system*	line	second
data	line of symmetry	sequence
decimal	line plot	simplest form*
decimal fraction	line segment	simplify
decimal notation	line symmetric figures	square unit
decimal point	liter	standard form*
decompose	lowest terms	subtract
decrease*	mass	sum
degree* (angle measure)	mean*	tenth*
denominator*	median*	time interval
digit	meter	two-dimensional
difference	metric system*	unit fraction
distributive property	mile	unlike denominators
divide	milliliter	variable*
dividend*	millimeter	vertex (vertices)
divisor*	minute	volume

endpoint	mixed number*	weight
equal	mode*	whole numbers
equation	multiple*	word form*
equivalent fractions*	multiplicative comparison	yard
estimate	multiply	zero property of multiplication

(* denotes mastery of vocabulary word)

Grade 5

MA 5.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 5.1.1	Number System: Students will represent and show relationships among positive rational numbers.
*	MA 5.1.1.a Demonstrate multiple equivalent representations for whole numbers and decimals through the thousandths place (e.g., 3.125 is 3 + .1 + .02 + .005)
	Write numbers in expanded form
*	MA 5.1.1.b Compare and order whole numbers, fractions, and decimals through the thousandths place
*	MA 5.1.1.c Identify and name fractions in their simplest form and find common denominators for fractions
	Fraction means divide
	Multiply fractions
*	MA 5.1.1.d Recognize and generate equivalent forms of commonly used fractions, decimals, and percents (e.g., one third, one fourth, one half, two thirds, three fourths)
*	MA 5.1.1.e Classify a number as prime or composite
*	MA 5.1.1.f Identify factors and multiples of any whole number
	Comparing multiplication factors
	MA 5.1.1.g Round whole numbers and decimals to any given place
MA 5.1.2	Operations: Students will demonstrate the meaning of arithmetic operations with whole numbers.
	MA 5.1.2.a Use words and symbols to explain the meaning of the

	identity properties for addition and multiplication
	MA 5.1.2.b Use words and symbols to explain the meaning of the commutative and associative properties of addition and multiplication
*	MA 5.1.2.c Use words and symbols to explain the distributive property of multiplication over addition (e.g., 5 (y + 2) = 5y + 5 x 2)
MA 5.1.3	Computation: Students will compute fluently and accurately using appropriate strategies and tools.
*	MA 5.1.3.a Add and subtract positive rational numbers (e.g., proper and improper fractions, mixed numbers, fractions with common and uncommon denominators, decimals through the thousandths place)
*	MA 5.1.3.b Select, apply and explain the appropriate method of computation when problem solving (e.g., models, mental computation, paper-pencil, technology)
*	MA 5.1.3.c Multiply decimals
	Multiply fractions and mixed numbers
	Multiply and divide by powers of ten (exponents)
	MA 5.1.3.d Divide a decimal by a whole number
	Divide fractions and mixed numbers
MA 5.1.4	Estimation: Students will estimate and check reasonableness of answers using appropriate strategies and tools.
*	MA 5.1.4.a Estimate the sums and differences of positive rational numbers to check the reasonableness of such results
	Estimate products when multiplying two-digit numbers
	Make and interpret line plots
MA 5.2 Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 5.2.1	Characteristics: Students will describe relationships among two-dimensional shapes and three-dimensional objects.

*	MA 5.2.1.a Identify the number of edges, faces, and vertices of triangular and rectangular prisms
	MA 5.2.1.b Justify congruence of two-dimensional shapes
	MA 5.2.1.c Justify the classification of two-dimensional shapes (e.g., triangles by angles and sides)
*	MA 5.2.1.d Identify degrees on a circle (e.g., 45, 90, 180, 270, 360)
MA 5.2.2	Coordinate Geometry: Students will identify locations using coordinate geometry.
*	MA 5.2.2.a Plot the location of an ordered pair in the first quadrant
	Understand coordinate planes
MA 5.2.3	Transformations: Students will identify and use simple transformations.
	MA 5.2.3.a Perform one-step transformations on two dimensional shapes (e.g., translation, rotation, reflection, of 90, 180, and 270)
MA 5.2.4	Spatial Modeling: Students will create and use geometric models to solve problems.
	MA 5.2.4.a Build or sketch a geometric model to solve a problem
	MA 5.2.4.b Sketch congruent shapes
	MA 5.2.4.c Build rectangular prisms using cubes
MA 5.2.5	Measurement: Students will apply appropriate procedures, tools, and formulas to determine measurements using customary and metric units.
	Convert measures from meters to centimeters
	MA 5.2.5.a Select and use appropriate tools to measure perimeter and angles

*	MA 5.2.5.b Identify correct unit (customary or metric) to the measurement situation (e.g., distance from home to school; measure length of a room)
	MA 5.2.5.c Estimate and measure length with customary units to the nearest ¼ inch
	MA 5.2.5.d Measure capacity/volume with customary units
	Recognize volume as an attribute of a solid
	Solve problems with volume
	Find volume of a rectangular prism
	Apply formulas to find volume
	Interface area of a solid
	MA 5.2.5.e Measure weight (mass) and temperature using metric units
*	MA 5.2.5.f Determine the area of rectangles and squares
	Find area using fractions
MA 5.3 Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 5.3.1	Relationships: Students will represent, analyze, and generalize relationships.
	MA 5.3.1.a Describe, extend, apply rules, and make generalizations about numeric, and geometric patterns
	MA 5.3.1.b Create and analyze numeric patterns using words, tables, and graphs
	MA 5.3.1.c Communicate relationships using expressions and equations
MA 5.3.2	Modeling in Context: Students will create, use, and compare models

	representing mathematical situations.
*	MA 5.3.2.a Model situations that involve the addition, subtraction, and multiplication of positive rational numbers using words, graphs, and tables
	MA 5.3.2.b Represent a variety of quantitative relationships using tables and graphs
	MA 5.3.2.c Compare different models to represent mathematical situations
MA 5.3.3	Procedures: Students will apply properties of simple positive rational numbers to solve one-step equations.
	MA 5.3.3.a Explain the addition property of equality (e.g., if a = b, then a + c = b + c)
*	MA 5.3.3.b Use symbolic representations of the associative property (e.g., (2 + 3) + 4 = 2 + (3 + n), (2 * 3) * 4 = 2 * (3 * n))
*	MA 5.3.3.c Evaluate numerical expressions by using parentheses with respect to order of operations (e.g., 6 + (3 * 5))
*	MA 5.3.3.d Evaluate simple algebraic expressions involving addition and subtraction
*	MA 5.3.3.e Solve one-step addition and subtraction equations involving common positive rational numbers
	MA 5.3.3.f Identify and explain the properties of equality used in solving one-step equations involving common positive rational numbers
	Write numerical expressions
MA 5.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 5.4.1	Display and Analysis: Students will organize, display, compare, and interpret data.

	Collect data using observations and surveys
*	MA 5.4.1.a Represent data using line plots
*	MA 5.4.1.b Represent the same set of data in different formats (e.g., table, pictographs, bar graphs, line plots)
	Describe and compare data sets
*	MA 5.4.1.c Draw conclusions based on a set of data
	MA 5.4.1.d Find the mean, median, mode, and range for a set of whole numbers
	MA 5.4.1.e Generate questions and answers from data sets and their graphical representations
MA 5.4.2	Predictions and Inferences: Students will construct predictions based on data.
	MA 5.4.2.a Make predictions based on data to answer questions from tables, bar graphs, and line plots
MA 5.4.3	Probability: Students will determine theoretical probabilities.
	MA 5.4.3.a Perform and record results of probability experiments
*	MA 5.4.3.b Generate a list of possible outcomes for a simple event
*	MA 5.4.3.c Explain that the likelihood of an event that can be represented by a number from 0 (impossible) to 1 (certain)

Fifth Grade Mathematics Vocabulary

Acute angle*	Inch	Protractor*
Addend	Increasing	Pyramid
Addition	Intersecting	Quadrant*
Area	Inverse operations*	Quadrilateral
Array	Isosceles triangle*	Quart

Average*	Key	Quarter inch*
Bar graph	Kilogram*	Quotient
Base*	Kite	Rectangle
Braces	Length	Rectangular prism
Brackets	Less than	Rectilinear figure
Centimeter	Liquid volume	Remainder
Composite	Liter	Rhombus*
Cone	Lowest terms*	Right angle
Congruent	Mass*	Right triangle*
Coordinate plane*	Maximum*	Round
Cube	Mixed number	Rule sequence
Cubic units*	Multiple	Scale*
Curved surface	Multiplication	Scalene triangle*
Customary unit	Net*	Similar
Cylinder	Nonstandard unit*	Sphere
Data	Numerator	Square
Decimal point	Numerical expression*	Square centimeter*
Decreasing	Obtuse angle*	Square foot*
Denominator	Octagon*	Square inch*
Difference	One-eighth	Square meter*
Dividend	One-fourth	Square unit*
Division	One-half	Standard unit*
Divisor	One-sixth	Strategy
Edge*	One-third	Subtraction
Equation	Operations*	Sum
Equilateral triangle*	Ordered pair*	Surface area*
Equivalent fractions	Origin*	Symmetrical
Estimate	Parallel	Tenth
Expanded form	Parallelogram*	Term
Exponent*	Parentheses	Thousandth*
Face	Pattern	Trapezoid*
Factor	Pentagon*	Two-dimensional
First quadrant	Percentage*	Unit fraction
Flat surface	Perimeter	Unknown
Formula*	Perpendicular	Unlike denominator
Gallon	Picture graph line plot	Vertices
Gram*	Pint	Volume
Greater than	Place value	Whole
Half inch	Polygon	Width metric unit
Height	Power of ten*	x-axis*
Hexagon*	Prime	y-axis*
Hundredth	Product	

(* denotes mastery of vocabulary word)

Grade 6

MA 6.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 6.1.1	Number System: Students will represent and show relationships among positive rational numbers and integers.
	MA 6.1.1.a Show equivalence among common fractions, decimals and percents
*	MA 6.1.1.b Compare and order positive and negative integers
	MA 6.1.1.c Identify integers less than 0 on a number line
*	MA 6.1.1.d Represent large numbers using exponential notation (e.g., $1{,}000 = 10^3$)
*	MA 6.1.1.e Identify the prime factorization of numbers (e.g., 12 = 2 x 2 x 3 or 2^2 x 3)
	MA 6.1.1.f Classify numbers as natural, whole, or integer
	Understand the concept of a unit rate a/b associated with a ratio a:b with b=0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."
	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
MA 6.1.2	Operations: Students will demonstrate the meaning of arithmetic operations with positive fractions and decimals.
*	MA 6.1.2.a Use drawings, words, and symbols to explain the meaning of addition and subtraction of fractions

*	MA 6.1.2.b Use drawings, words, and symbols to explain the meaning of addition and subtraction of decimals
MA 6.1.3	Computation: Students will compute fluently and accurately using appropriate strategies and tools.
*	MA 6.1.3.a Multiply and divide positive rational numbers
*	MA 6.1.3.b Select and apply the appropriate method of computation when problem solving (e.g., models, mental computation, paper-pencil, technology, divisibility rules)
MA 6.1.4	Estimation: Students will estimate and check reasonableness of answers using appropriate strategies and tools.
*	MA 6.1.4.a Use appropriate estimation methods to check the reasonableness of solutions for problems involving positive rational numbers
MA 6.2 Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 6.2.1	Characteristics: Students will compare and contrast properties among two-dimensional shapes and among three-dimensional objects.
	MA 6.2.1.a Justify the classification of three dimensional objects
MA 6.2.2	Coordinate Geometry: Students will label points using coordinate geometry.
*	MA 6.2.2.a Identify the ordered pair of a plotted point in the coordinate plane
MA 6.2.3	Transformations: Students will use and describe results of transformations on geometric shapes.
	MA 6.2.3.a Perform and describe positions and orientation of shapes under single transformations (translation, rotation, reflection) not on a coordinate plane

MA 6.2.4	Spatial Modeling: Students will use visualization of geometric models to solve problems.
*	MA 6.2.4.a Identify two-dimensional drawings of three-dimensional objects
MA 6.2.5	Measurement: Students will apply appropriate procedures, tools, and formulas to determine measurements.
	MA 6.2.5.a Estimate and measure length with customary and metric units to the nearest 1/16 inch and mm
	MA 6.2.5.b Measure volume/capacity using the metric system
	MA 6.2.5.c Convert length, weight (mass), and liquid capacity from one unit to another within the same system
*	MA 6.2.5.d Determine the perimeter of polygons
*	MA 6.2.5.e Determine the area of parallelograms and triangles
*	MA 6.2.5.f Determine the volume of rectangular prisms
MA 6.3 Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 6.3.1	Relationships: Students will represent, analyze, and use relationships to make generalizations.
*	MA 6.3.1.a Describe and create simple algebraic expressions (e.g., one operation, one variable) from words and tables
*	MA 6.3.1.b Use a variable to describe a situation with an equation (e.g., one-step, one variable)
	MA 6.3.1.c Identify relationships as increasing, decreasing, or constant

MA 6.3.2	Modeling in Context: Students will create, use, and interpret models of quantitative relationships.
*	MA 6.3.2.a Model contextualized problems using various representations (e.g., graphs, tables)
	MA 6.3.2.b Represent a variety of quantitative relationships using symbols and words
MA 6.3.3	Procedures: Students will apply properties to solve equations.
	MA 6.3.3.a Explain the multiplication property of equality (e.g., if a = b, then ac = bc)
*	MA 6.3.3.b Evaluate numerical expressions containing multiple operations with respect to order of operations (e.g., 2 + 4 x 5)
*	MA 6.3.3.c Evaluate simple algebraic expressions involving multiplication and division
*	MA 6.3.3.d Solve one-step equations involving positive rational numbers
*	MA 6.3.3.e Identify and explain the properties of equality used in solving one-step equations (e.g., addition, subtraction, division)
MA 6.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 6.4.1	Display and Analysis: Students will organize, display, compare, and interpret data.
	MA 6.4.1.a Represent data using stem and leaf plots, histograms, and frequency charts
*	MA 6.4.1.b Compare and interpret data sets and their graphical representations
*	MA 6.4.1.c Find the mean, median, mode, and range for a set of data

	MA 6.4.1.d Compare the mean, median, mode, and range from two sets of data
MA 6.4.2	Predictions and Inferences: Students will construct predictions based on data.
	MA 6.4.2.a Make predictions based on data and create questions to further investigate the quality of the predictions
MA 6.4.3	Probability: Students will apply basic concepts of probability.
	MA 6.4.3.a Describe the theoretical probability of an event using a fraction, percentage, decimal, or ratio
*	MA 6.4.3.b Compute theoretical probabilities for independent events
*	MA 6.4.3.c Find experimental probability for independent events

Sixth Grade Mathematics Vocabulary

absolute value*	expression	polygon
acute triangle	factor	positive numbers*
addend	first quartile	prism*
additive identity property of zero	formula	product
additive inverses	fraction	proportion*
algebraic expression*	frequency	pyramid
Algorithm	gap	quadrants
Area	geometry*	quadrilateral
associative property of addition	geometric solid	quantity*
associative property of multiplication	graph	quotient
Attribute	greater than	range
axis (pl. axes)*	greatest common factor*	rate*
Bar graph	height	ratio*
base of a polygon	histogram	rational number*
box plot	improper fraction	reciprocals*
capacity	independent variable*	rectangle
cluster	inequality*	right rectangular prism
coefficient	infinite*	right triangle
common denominator	integers*	scalene triangle

common factor*	interquartile range	signed number
common multiple*	intersecting lines	solid figure
commutative property of addition	inverse operations	spread
compose commutative property of multiplication	isosceles triangle	statistical variability
congruent	kilometer	statistics*
constant*	least common multiple*	substitution*
constant speed	less than	subtrahend
coordinate cubic unit	line of symmetry	sum
coordinate pair	line graph	surface area
coordinate plane	line plot	table*
coordinate system	lower extreme	tape diagram
coordinates*	magnitude	term*
cube	maximum	third quartile
cubic unit	mead	three-dimensional
customary system	mean	triangular prism*
data	mean absolute deviation	triangular pyramid*
Decimal number and places	measure of center	unit cube
diameter	measure of variation	unite rate
decimeter	median	upper extreme
decompose	metric system	value*
denominator	minimum	variable
dependent variable*	minuend	vertex (vertices)
diagram*	mode	volume
difference	mixed number	whole numbers
distribution*	multiple	x-axis
distributive property	multiplicative identity property of one	x-coordinate*
dividend	multiplicative inverses*	y-axis
divisibility	negative numbers*	y-coordinate*
divisor	net	
dot plot	number line	
double number line	numerator	
equation	numerical expression	
equilateral triangle	obtuse triangle	
equivalent	opposite	
equivalent ratio*	order of operations	
evaluate*	ordered pair	
exponent	origin	
exponent expression	outlier	
	percent	
	plot*	

Grade 7

MA 7.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 7.1.1	Number System: Students will represent and show relationships among rational numbers.
*	MA 7.1.1.a Show equivalence among fractions, decimals, and percents
*	MA 7.1.1.b Compare and order rational numbers (e.g., fractions, decimals, percents)
*	MA 7.1.1.c Represent large numbers using scientific notation
	MA 7.1.1.d Classify numbers as natural, whole, integer, or rational
	MA 7.1.1.e Find least common multiple and greatest common divisor given two numbers
MA 7.1.2	Operations: Students will demonstrate the meaning of arithmetic operations with positive fractions, decimals, and integers.
	MA 7.1.2.a Use drawings, words, and symbols to explain the meaning of multiplication and division of fractions (e.g., 2/3 x 6 as two-thirds of six, or 6 x 2/3 as 6 groups of two-thirds, or 6 ÷ 2/3 as how many two-thirds there are in six.)
	MA 7.1.2.b Use drawings, words, and symbols to explain the meaning of multiplication and division of decimals
	MA 7.1.2.c Use drawings, words, and symbols to explain the addition and subtraction of integers
MA 7.1.3	Computation: Students will compute fluently and accurately using appropriate strategies and tools.
*	MA 7.1.3.a Compute accurately with integers

*	MA 7.1.3.b Select, apply, and explain the method of computation when problem solving using integers and positive rational numbers (e.g., models, mental computation, paper-pencil, technology, divisibility rules)
*	MA 7.1.3.c Solve problems involving percent of numbers (e.g., percent of, % increase, % decrease)
MA 7.1.4	Estimation: Students will estimate and check reasonableness of answers using appropriate strategies and tools.
*	MA 7.1.4.a Use estimation methods to check the reasonableness of solutions for problems involving integers and positive rational numbers
MA 7.2 Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 7.2.1	Characteristics: Students will describe, compare, and contrast characteristics, properties, and relationships of geometric shapes and objects.
	MA 7.2.1.a Identify and describe similarity of two-dimensional shapes using side and angle measurements
	MA 7.2.1.b Name line, line segment, ray, and angle (e.g., $\overleftrightarrow{\mathbf{AB}}$, $\overrightarrow{\mathbf{PR}}$, $<$ $\mathbf{LMN}$)
MA 7.2.2	Coordinate Geometry: Students will specify locations and describe relationships using coordinate geometry.
*	MA 7.2.2.a Plot the location of an ordered pair in the coordinate plane
	MA 7.2.2.b Identify the quadrant of a given point in the coordinate plane
*	MA 7.2.2.c Find the distance between points along horizontal and vertical lines of a coordinate plane (e.g., what is the distance between (0, 3) and (0, 9))

MA 7.2.3	Transformations: Students will use transformations and symmetry to analyze geometric shapes.
	MA 7.2.3.a Identify lines of symmetry for a reflection
*	MA 7.2.3.b Perform and describe positions and orientation of shapes under a single transformation (e.g., translation, rotation, reflection) on a coordinate plane
MA 7.2.4	Spatial Modeling: Students will use visualization to create geometric models in solving problems.
	MA 7.2.4.a Identify the shapes that make up the three-dimensional object
	MA 7.2.4.b Create two-dimensional representations of three-dimensional objects to visualize and solve problems (e.g., perspective drawing of surface area)
	MA 7.2.4.c Draw angles to given degree
MA 7.2.5	Measurement: Students will select and apply appropriate procedures, tools, and formulas to determine measurements.
	MA 7.2.5.a Measure angles to the nearest degree
*	MA 7.2.5.b Determine the area of trapezoids and circles, and the circumference of circles
	MA 7.2.5.c Recognize the inverse relationship between the size of a unit and the number of units used when measuring
MA 7.3 Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 7.3.1	Relationships: Students will represent and analyze relationships using algebraic symbols.
*	MA 7.3.1.a Describe and create algebraic expressions from words, tables, and graphs

*	MA 7.3.1.b Use a variable to describe a situation with an inequality (e.g., one-step, one variable)
	MA 7.3.1.c Recognize and generate equivalent forms of simple algebraic expressions
	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
MA 7.3.2	Modeling in Context: Students will create, use, and interpret models of quantitative relationships.
*	MA 7.3.2.a Model contextualized problems using various representations (e.g., one-step/variable expressions, one-step/variable equations)
	MA 7.3.2.b Represent a variety of quantitative relationships using algebraic expressions and one-step equations
MA 7.3.3	Procedures: Students will apply properties to solve equations and inequalities.
	MA 7.3.3.a Explain additive inverse of addition (e.g., 7 + -7 = 0)
	MA 7.3.3.b Use symbolic representation of the distributive property (e.g., 2(x + 3) = 2x + 6)
*	MA 7.3.3.c Given the value of the variable(s), evaluate algebraic expressions with respect to order of operations
*	MA 7.3.3.d Solve two-step equations involving integers and positive rational numbers
*	MA 7.3.3.e Solve one-step inequalities involving positive rational numbers
	MA 7.3.3.f Identify and explain the properties used in solving two-step equations (e.g., addition, subtraction, multiplication and division)
	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

MA 7.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 7.4.1	Display and Analysis: Students will formulate questions that can be addressed with data and then organize, display, and analyze the relevant data to answer their questions.
*	MA 7.4.1.a Analyze data sets and interpret their graphical representations
*	MA 7.4.1.b Find and interpret mean, median, mode, and range for sets of data
	MA 7.4.1.c Explain the difference between a population and a sample
	MA 7.4.1.d List biases that may be created by various data collection processes
	MA 7.4.1.e Formulate a question about a characteristic within one population that can be answered by simulation or a survey
MA 7.4.2	Predictions and Inferences: Students will evaluate predictions and make inferences based on data.
	MA 7.4.2.a Determine if data collected from a sample can be used to make predictions about a population
MA 7.4.3	Probability: Students will apply and interpret basic concepts of probability.
*	MA 7.4.3.a Find the probability of independent compound events (e.g., tree diagram, organized list)
*	MA 7.4.3.b Compare and contrast theoretical and experimental probabilities

Grade 8

MA 8.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 8.1.1	Number System: Students will represent and show relationships among real numbers.
*	MA 8.1.1.a Compare and order real numbers
	MA 8.1.1.b Demonstrate relative position of real numbers on the number line (e.g., square root of 2 is left of 1.5)
*	MA 8.1.1.c Represent small numbers using scientific notation
*	MA 8.1.1.d Classify numbers as natural, whole, integer, rational, irrational, or real
MA 8.1.2	Operations: Students will demonstrate the meaning of arithmetic operations with integers.
	MA 8.1.2.a Use drawings, words, and symbols to explain the meaning of addition, subtraction, multiplication, and division of integers.
	MA 8.1.2.b Use words and symbols to explain the zero property of multiplication (e.g., if ab = 0 then a or b or both must be zero)
	MA 8.1.2.c Use words and symbols to explain why division by zero is undefined
MA 8.1.3	Computation: Students will compute fluently and accurately using appropriate strategies and tools.
*	MA 8.1.3.a Compute accurately with rational numbers
*	MA 8.1.3.b Evaluate expressions involving absolute value of integers

	MA 8.1.3.c Calculate squares of integers, the square roots of perfect squares, and the square roots of whole numbers using technology
*	MA 8.1.3.d Select, apply, and explain the method of computation when problem solving using rational numbers (e.g., models, mental computation, paper-pencil, technology, divisibility rules)
*	MA 8.1.3.e Solve problems involving ratios and proportions (e.g., $\frac{x}{5} = \frac{10}{17}$)
MA 8.1.4	Estimation: Students will estimate and check reasonableness of answers using appropriate strategies and tools.
*	MA 8.1.4.a Use estimation methods to check the reasonableness of solutions for problems involving rational numbers
MA 8.2 Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 8.2.1	Characteristics: Students will describe, compare, and contrast characteristics, properties, and relationships of geometric shapes and objects.
	MA 8.2.1.a Identify and describe similarity of three-dimensional objects
	MA 8.2.1.b Compare and contrast relationships between similar and congruent objects
*	MA 8.2.1.c Identify geometric properties of parallel lines cut by a transversal and related angles (e.g., perpendicular and parallel lines with transversals) and angles (e.g., corresponding, alternate interior, alternate exterior)
*	MA 8.2.1.d Identify pairs of angles (e.g., adjacent, complementary, supplementary, vertical)
*	MA 8.2.1.e Examine the relationships of the interior angles of a triangle (e.g., the sum of the angles is 180 degrees)
MA 8.2.2	Coordinate Geometry: Students will specify locations and describe relationships using coordinate geometry.

*	MA 8.2.2.a Use coordinate geometry to represent and examine the properties of rectangles and squares using horizontal and vertical segments
MA 8.2.3	Transformations: Students will perform transformations and use them to analyze the orientation and size of geometric shapes.
	MA 8.2.3.a Identify the similarity of dilated shapes
	MA 8.2.3.b Perform and describe positions and sizes of shapes under dilations (e.g., scale factor, ratios)
MA 8.2.4	Spatial Modeling: Students will use visualization, spatial reasoning, and geometric modeling to solve problems.
	MA 8.2.4.a Draw geometric objects with specified properties (e.g., parallel sides, number of sides, angle measures, number of faces)
MA 8.2.5	Measurement: Students will select and apply appropriate procedures, tools, and formulas to determine measurements.
	MA 8.2.5.a Use strategies to find the perimeter and area of complex shapes
	MA 8.2.5.b Determine surface area and volume of three-dimensional objects (e.g., rectangular prisms, cylinders)
*	MA 8.2.5.c Apply the Pythagorean theorem to find missing lengths in right triangles and to solve problems
*	MA 8.2.5.d Use scale factors to find missing lengths in similar shapes
	MA 8.2.5.e Convert between metric and standard units of measurement, given conversion factors (e.g., meters to yards)
	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for

	similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.
MA 8.3 Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 8.3.1	Relationships: Students will represent and analyze relationships using algebraic symbols.
	MA 8.3.1.a Represent and analyze a variety of patterns with tables, graphs, words, and algebraic equations
*	MA 8.3.1.b Describe relationships using algebraic expressions, equations, and inequalities (e.g., two-step, one variable)
	MA 8.3.1.c Identify constant slope from tables and graphs
MA 8.3.2	Modeling in Context: Students will create, use, and interpret models of quantitative relationships.
*	MA 8.3.2.a Model contextualized problems using various representations (e.g., two-step/one variable equations)
	MA 8.3.2.b Represent a variety of quantitative relationships using algebraic expressions and two-step/one variable equations
MA 8.3.3	Procedures: Students will apply properties to solve equations and inequalities.
	MA 8.3.3.a Explain the multiplicative inverse (e.g., 4 * ¼= 1)
*	MA 8.3.3.b Evaluate numerical expressions containing whole number exponents (e.g., if x = 4, then $(x + 3)^2 + 5x = ?$)
*	MA 8.3.3.c Solve multi-step equations involving rational numbers

*	MA 8.3.3.d Solve two-step inequalities involving rational numbers
	MA 8.3.3.e Identify and explain the properties used in solving two-step inequalities and multi-step equations
MA 8.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 8.4.1	Display and Analysis: Students will formulate questions that can be addressed with data, and then organize, display, and analyze the relevant data to answer their questions.
	MA 8.4.1.a Represent data using circle graphs and box plots with and without the use of technology
*	MA 8.4.1.b Compare characteristics between sets of data or within a given set of data
	MA 8.4.1.c Find, interpret, and compare measures of central tendency (mean, median, mode) and the quartiles for sets of data
*	MA 8.4.1.d Select the most appropriate unit of central tendency for sets of data
*	MA 8.4.1.e Identify misrepresentation and misinterpretation of data represented in circle graphs and box plots
MA 8.4.2	Predictions and Inferences: Students will evaluate predictions and make inferences based on data.
	MA 8.4.2.a Evaluate predictions to formulate new questions and plan new studies
	MA 8.4.2.b Compare and contrast two sets of data to make inferences

MA 8.4.3	Probability: Students will apply and interpret basic concepts of probability.
*	MA 8.4.3.a Identify complementary events and calculate their probabilities
*	MA 8.4.3.b Compute probabilities for independent compound events

Grade 12

MA 12.1 Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 12.1.1	Number System: Students will represent and show relationships among complex numbers.
	MA 12.1.1.a Demonstrate multiple equivalent forms of irrational numbers (e.g., $\sqrt{8} = 8^{1/2} = 2\sqrt{2}$)
	MA 12.1.1.b Compare, contrast and apply the properties of numbers and the real number system, including rational, irrational, imaginary, and complex numbers
MA 12.1.2	Operations: Students will demonstrate the meaning and effects of arithmetic operations with real numbers.
	MA 12.1.2.a Use drawings, words, and symbols to explain the effects of such operations as multiplication and division, and computing positive powers and roots on the magnitude of quantities (e.g., if you take the square root of a number, will the result always be smaller than the original number? (e.g., $\sqrt{1/4} = 1/2$))
	MA 12.1.2.b Use drawings, words, and symbols to explain that the distance between two numbers on the number line is the absolute value of their difference
MA 12.1.3	Computation: Students will compute fluently and accurately using appropriate strategies and tools.
*	MA 12.1.3.a Compute accurately with real numbers.
*	MA 12.1.3.b Simplify exponential expressions (e.g., powers of -1, 0, ½, $3^2 * 3^2 = 3^4$)
	MA 12.1.3.c Multiply and divide numbers using scientific notation
	MA 12.1.3.d Select, apply, and explain the method of computation when problem solving using real numbers (e.g., models, mental computation, paper-pencil, or technology)
	Use the relation i2=-1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers (Algebra II)
MA 12.1.4	Estimation: Students will estimate and check reasonableness of answers using appropriate strategies and tools.

*	MA 12.1.4.a Use estimation methods to check the reasonableness of real number computations and decide if the problem calls for an approximation or an exact number (e.g., 10 π (pi) is approximately 31.4, square and cube roots)
	MA 12.1.4.b Distinguish relevant from irrelevant information, identify missing information and either find what is needed or make appropriate estimates
MA 12.2 Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 12.2.1	Characteristics: Students will analyze characteristics, properties, and relationships among geometric shapes and objects.
	MA 12.2.1.a Identify and explain the necessity of and give examples of definitions and theorems
	MA 12.2.1.b Analyze properties and relationships among classes of two and three dimensional geometric objects using inductive reasoning and counterexamples
	MA 12.2.1.c State and prove geometric theorems using deductive reasoning (e.g., parallel lines with transversals, congruent triangles, similar triangles)
*	MA 12.2.1.d Apply geometric properties to solve problems (e.g., parallel lines, line transversals, similar triangles, congruent triangles, proportions)
*	MA 12.2.1.e Identify and apply right triangle relationships (e.g., sine, cosine, tangent, special right triangles, converse of Pythagorean Theorem)
	MA 12.2.1.f Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true
	MA12.2.1.g Know the definitions and basic properties of a circle and use them to prove basic theorems and solve problems
MA 12.2.2	Coordinate Geometry: Student will use coordinate geometry to analyze and describe relationships in the coordinate plane.
*	MA 12.2.2.a Use coordinate geometry to analyze geometric situations (e.g., parallel lines, perpendicular lines, circle equations)

	MA 12.2.2.b Apply the midpoint formula
*	MA 12.2.2.c Apply the distance formula
*	MA 12.2.2.d Prove special types of triangles and quadrilaterals (e.g., right triangles, isosceles trapezoid, parallelogram, rectangle, square)
MA 12.2.3	Transformations: Students will apply and analyze transformations.
	MA 12.2.3.a Explain and justify the effects of simple transformations on the ordered pairs of two-dimensional shapes
	MA 12.2.3.b Perform and describe multiple transformations
MA 12.2.4	Spatial Modeling: Students will use visualization, spatial reasoning, and geometric modeling to solve problems.
	MA 12.2.4.a Sketch and draw appropriate representations of geometric objects using ruler, protractor, or technology
*	MA 12.2.4.b Use geometric models to visualize, describe, and solve problems (e.g., find the height of a tree; find the amount of paint needed for a room; scale model.
MA 12.2.5	Measurement: Students will apply the units, systems, and formulas to solve problems.
	MA 12.2.5.a Use strategies to find surface area and volume of complex objects
	MA 12.2.5.b Apply appropriate units and scales to solve problems involving measurement
	MA 12.2.5.c Convert between various units of area and volume, such as square feet to square yards
*	MA 12.2.5.d Convert equivalent rates (e.g., feet/second to miles/hour)

	MA 12.2.5.e Find arc length and area of sectors of a circle
	MA 12.2.5.f Determine surface area and volume of three-dimensional objects (e.g., spheres, cones, pyramids)
	MA12.2.5.g Know that the effect of a scale factor k on length, area and volume is to multiply each by k, k^2 and k^3, respectively
MA 12.3 Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 12.3.1	Relationships: Students will generalize, represent, and analyze relationships using algebraic symbols.
	Nonlinear functions include: quadratic, absolute, absolute value, square route, exponential
*	MA 12.3.1.a Represent, interpret, and analyze functions with graphs, tables, and algebraic notation and convert among these representations (e.g., linear, non-linear)
	MA 12.3.1.b Identify domain and range of functions represented in either symbolic or graphical form (e.g., linear, non-linear)
*	MA 12.3.1.c Identify the slope and intercepts of a linear relationship from an equation or graph
*	MA 12.3.1.d Identify characteristics of linear and non-linear functions
	MA 12.3.1.e Graph linear and non-linear functions
*	MA 12.3.1.f Compare and analyze the rate of change by using ordered pairs, tables, graphs, and equations
	MA 12.3.1.g Graph and interpret linear inequalities

	MA 12.3.1.h Represent, interpret, and analyze functions and their inverses
	MA 12.3.1.i Determine if a relation is a function
MA 12.3.2	Modeling in Context: Students will model and analyze quantitative relationships.
	Contextualized problem – a mathematical situation placed in a particular context (e.g. Using words, diagrams, tables, drawings, etc.)
	MA 12.3.2.a Model contextualized problems using various representations (e.g., graphs, tables, one variable equalities, one variable inequalities, linear equations in slope intercept form, inequalities in slope intercept form, system of linear equations with two variables)
*	MA 12.3.2.b Represent a variety of quantitative relationships using linear equations and one variable inequalities
	MA 12.3.2.c Analyze situations to determine the type of algebraic relationship (e.g., linear, nonlinear)
	MA 12.3.2.d Model contextualized problems using various representations for non-linear functions (e.g., quadratic, exponential, square root, and absolute value)
MA 12.3.3	Procedures: Students will represent and solve equations and inequalities.
	MA 12.3.3.a Explain/apply the reflexive, symmetric, and transitive properties of equality
*	MA 12.3.3.b Simplify algebraic expressions involving exponents (e.g., $(3x^4)^2$)
*	MA 12.3.3.c Add and subtract polynomials
*	MA 12.3.3.d Multiply and divide polynomials (e.g., divide $x^3 – 8$ by $x – 2$, divide $x^4 – 5x^3 – 2x$ by x^2)

	MA 12.3.3.e Factor polynomials
*	MA 12.3.3.f Identify and generate equivalent forms of linear equations
	MA 12.3.3.g Solve linear equations and inequalities including absolute value
	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (Algebra I & II)
	MA 12.3.3.h Identify and explain the properties used in solving equations and inequalities
	MA 12.3.3.i Solve quadratic equations (e.g., factoring, graphing, quadratic formula)
	MA 12.3.3.j Add, subtract, and simplify rational expressions
	MA 12.3.3.k Multiply, divide, and simplify rational expressions
	MA 12.3.3.l Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified values of their variables
	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (Algebra I & II)
	MA 12.3.3.m Derive and use the formulas for the general term and summation of finite arithmetic and geometric series
	MA 12.3.3.n Combine functions by composition, as well as by addition, subtraction, multiplication, and division
	MA 12.3.3.o Solve an equation involving several variables for one variable in terms of the others
	MA 12.3.3.p Analyze and solve systems of two linear equations in two variables algebraically and graphically

	MATRIX MATH (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (Algebra II)
	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. (Algebra II)
	(+) Add, subtract, and multiply matrices of appropriate dimensions. (Algebra II)
	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. (Algebra II)
	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. (Algebra II)
	(+) Represent a system of linear equations as a single matrix equation in a vector variable. (Algebra II)
	(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x or greater).(Algebra II)
MA 12.4 Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.	
MA 12.4.1	Display and Analysis: Students will formulate a question and design a survey or an experiment in which data is collected and displayed in a variety of formats, then select and use appropriate statistical methods to analyze the data.
	MA 12.4.1.a Interpret data represented by the normal distribution and formulate conclusions
	MA 12.4.1.b Compute, identify, and interpret measures of central tendency (mean, median, mode) when provided a graph or data set
	MA 12.4.1.c Explain how sample size and transformations of data affect measures of central tendency
*	MA 12.4.1.d Describe the shape and determine spread (variance, standard deviation) and outliers of a data set

	MA 12.4.1.e Explain how statistics are used or misused in the world
	MA 12.4.1.f Create scatter plots, analyze patterns, and describe relationships in paired data
	MA 12.4.1.g Explain the impact of sampling methods, bias, and the phrasing of questions asked during data collection and the conclusions that can rightfully be made
	MA 12.4.1.h Explain the differences between randomized experiment and observational studies
MA 12.4.2	Predictions and Inferences: Students will develop and evaluate inferences to make predictions.
	MA 12.4.2.a Compare data sets and evaluate conclusions using graphs and summary statistics
	MA 12.4.2.b Support inferences with valid arguments
	MA 12.4.2.c Develop linear equations for linear models to predict unobserved outcomes using regression line and correlation coefficient
	MA 12.4.2.d Recognize when arguments based on data confuse correlation with causation
MA 12.4.3	Probability: Students will apply and analyze concepts of probability.
	MA 12.4.3.a Construct a sample space and a probability distribution
*	MA 12.4.3.b Identify dependent and independent events and calculate their probabilities
*	MA 12.4.3.c Use the appropriate counting techniques to determine the probability of an event (e.g., combinations, permutations)
*	MA 12.4.3.d Analyze events to determine if they are mutually exclusive

	MA 12.4.3.e Determine the relative frequency of a specified outcome of an event to estimate the probability of the outcome
MA: ADVANCED CONCEPTS **Students will communicate advanced mathematical concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.**	
Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (Advanced Math)	
Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (Advanced Math)	
Use the special triangles to determine geometrically the values of sine, cosine, tangent for **TT/3, TT/4 and TT/6,** and use the unit circle to express the values of sine, cosines, and tangent for x, **TT + x, and 2TT – x** in terms of their values for x, where x is any real number.	
Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* (Advance Math)	
Prove the Pythagorean identity **sin2(0) + cos2 (0)=1 and use it to find sin(0), cos(0), or tan(0) given sin(0), cos(0), or tan(0)** and the quadrant of the angle. (Advance Math)	
(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. (Advance Math)	
(+) Prove the Laws of Sines and Cosines and use them to solve problems. (Geometry).	
(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). (Geometry)	
Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (Geometry)	
Derive the equation of a parabola given a focus and directix. (Advanced Math)	
(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. (Advanced Math)	

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (Algebra I & II)
Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's Principle, and informal limit arguments. (Geometry)
(+) Find the conjugate of the complex number; use conjugates to find moduli and quotients of complex numbers. (Algebra I & II)
(+) Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x - 2i). (Advance Math)

Glossary

K - 6 Mathematics Glossary	
Vocabulary	Definition
Above	A preposition that indicates location of an object.
Absolute Value	The distance that a number is from zero on the number line.
Acute Angle	An angle that measures less than 90 degrees.
Acute Triangle	A triangle whose angles are all acute angles.
Add	Operation used to find the sum or two or more numbers.
Addend	Any number being added.
Addition	Operation used to find the sum of two or more numbers.
Additive Inverse	Two integers whose sum is zero.
After	Following in time or place.
Afternoon	The period between noon and evening.
Algebraic Expression	A mathematical phrase involving a variable or variables, numbers, and operations.
Algorithm	A step-by-step procedure for calculations.
Altitude	The height of an object. A segment from a vertex to the line containing the opposite side and perpendicular to that side.
Analog Clock	Shows time by pointing to numbers on a face.
Angle	Two rays that have the same endpoint.
Angle Measure	
AM	Time between midnight and noon.
Arc	A part of a circle that is defined by two endpoints.
Area	The measure of the interior region of a two-dimensional figure or the surface of a three-dimensional figure.
Array	An arrangement of objects in equal rows.
Associative Property **a) Addition** **b) Multiplicati on**	a) Addends can be regrouped and the sum remains the same. b) Factors can be regrouped and the product remains the same.
Attribute	A characteristic of an object such as color, shape, size, etc.
Bar Graph	A graph that uses the height or length of rectangles to compare data.
Base (of a polygon)	The side of a polygon to which the height is perpendicular.
Before	Earlier or sooner than.
Behind	A preposition that indicates location of an object.
Below	A preposition that indicates location of an object.
Benchmark Fractions	Fractions ¼, 1/3, ½, 2/3, ¾.
Beside	A preposition that indicates location of an object.
Between	A preposition that indicates location of an object.

Bigger	Of considerable size, number, quantity, magnitude, or extent.
Box (and Whisker) Plot	A diagram that divides a set of data into four parts using the median and quartiles.
Brackets	A form of parentheses.
By	A preposition that indicates location of an object.
Calendar	A system of organizing days for social, religious, commercial, or administrative purposes.
Capacity	The amount a container will hold.
Category	A collection of things sharing a common attribute.
Cent	1/100 of a dollar.
Centimeter	A metric unit of length.
Circle	The locus of all points in a plane equidistant from a given point called the center of the circle.
Classify	To sort into categories or to arrange into groups by attribute.
Clock	A tool used to tell time.
Closed Figure	A figure that can be traced with the same starting and stopping points, and without crossing or retracing any part of the figure.
Cluster	A small group or bunch of something.
Coefficient	The numerical factor of a term.
Color	The element of art that is produced when light, striking an object, is reflected back to the eye.
Common Denominator	A denominator that is the same in two or more fractions.
Common Factor	A factor that is the same for two or more numbers.
Common Multiple	A multiple that is the same for two or more numbers.
Commutative Property	Changing the order does not change the end result (applies to addition and multiplication).
Compare	To decide if one number is greater than, less than, or equal to another number.
Comparison Bars	< (less than), > (greater than), ≤ (less than or equal to), ≥ (greater than or equal to)
Compose	To put together basic elements.
Composite	A number greater than zero that has more than two different factors
Cone	A three-dimensional figure with one curved surface, one flat surface(usually a circle), one curved edge, and one vertex
Congruent	Having exactly the same size and shape.
Constant	A monomial that is a real number.
Coordinate Pair	A pair of points that represents a location in the coordinate plane.
Coordinate Plane	A two-dimensional system in which a location is described by its distances from two intersecting, usually perpendicular, straight lines called axes.
Coordinate System	The grid formed by the intersection of two number lines, the horizontal and vertical axes.
Corner	Where three or more edges meet.

Count	To name units of a group one by one in order to determine the total number. Counting tells how many things are in a set.
Count Up	To recite numerals in ascending order.
Cup	A customary unit of capacity equal to 8 ounces.
Cube	A right rectangular prism with all sides and angles equal.
Cubic Units	Measurement used when measuring volume.
Customary Unit	A unit of measurement used in the United States. The system includes units for measuring length, capacity, weight, and temperature.
Cylinder	A three-dimensional figure with two circular bases that is parallel and congruent.
Data	Information, especially numerical information.
Day	One of the 365 parts into which a year is divided.
Decimal	A number with one or more numbers to the right of the decimal point.
Decimal Fraction	A fraction that includes decimals.
Decimal Notation	The writing of numbers in a base-10 numeral system.
Decimal Point	A point used to separate whole numbers from parts of numbers.
Decompose	To separate into basic elements.
Decreasing	To grow progressively less (as in size, amount, number, or intensity).
Degree	Unit used to measure angles.
Denominator	The number below the fraction bar in a fraction.
Dependent Variable	The variable in a relation with a value that depends on the value of the independent variable.
Difference	The number that results from subtracting one number from another.
Different	Compare two or more objects or figures to find what is not the same. (e.g. different in size.)
Digit	Any one of the ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Digital Clock	Shows time by use of numbers rather than hands like a analog clock.
Dime	A coin with a value of ten cents.
Distributive Property	Multiplying a sum (or difference) by a number is the same as multiplying each number in the sum (or difference) by the number and adding (or subtracting) the products.
Dividend	The number to be divided.
Division	An operation that tells how many equal groups there are or how many are in each group. An operation that gives the quotient of two numbers.
Divisor	The number used to divide another number.
Dollar	Type of currency (money) used in the United States.
Dot Plot (Scatter Plot)	Two sets of data plotted as ordered pairs in a coordinate plane.
Double	Twice as much in size, strength, number, or amount.
Edge	A line segment where two faces meet in a solid figure.

Endpoint	A point at which a line segment or ray terminates.
Equal (groups, shares, parts)	Having the same amount.
Equation	A mathematical sentence with an equal sign.
Equilateral Triangle	A triangle whose sides all have the same length.
Equivalent	Having the same value.
Estimate	A number close to an exact amount; an estimate tells about how much or about how many.
Evaluate	To find the number that an algebraic expression names by replacing a variable with a given number. To solve.
Even	A number divisible by two.
Expanded Form	A way to write a number that shows the place value of each digit.
Exponent	The number that tells how many equal factors.
Expression	A combination of variables, numbers, and operation symbols that represents a mathematical relationship.
Face	A flat surface on a solid figure.
Fact Family (Related Facts)	A group of related facts using the same numbers.
Factor	An integer that divides evenly into another.
Factor Pairs	A pair of numbers whose product equals a given number.
Fewer Than	The smaller of two numbers being compared. (<)
Flat Surface	A plane horizontal surface with no depths.
Flip (Reflection)	A mirror image of a figure.
Foot	A customary unit of length equal to twelve inches.
Formula	An equation that states a rule.
Fraction	A way of describing a part of a whole or a group.
Function Table	A table of order pairs that follow a rule.
Future	The indefinite time yet to come.
Gallon	A customary unit of capacity equal to four quarts.
Gap	An opening in a solid structure or surface.
Gram	A metric unit of mass equal to 1,000 milligrams.
Graph	A representation of a set of objects where some pairs of the objects are connected by links. A picture representation of a set of data.
Greater Than	The larger of two numbers being compared. (>)
Greatest	Largest. Biggest.
Greatest Common Factor	The largest number that is a factor of two or more numbers.
Half Circle	Semicircle. Equal to 180°.
Half Dollar	Fifty-cent piece
Half Hour	Thirty minutes.
Half Inch	One of two equal parts of an inch.
Half Past	30 minutes past the hour.
Halves	Two equal parts.

Heavier	Having a weight that is greater than that of another object.
Height	The length of a segment from one vertex of a polygon perpendicular to the base.
Hexagon	A polygon with six faces.
Histogram	A graphical display that uses bars to display numerical data that have been organized into equal intervals.
Hour	A measure of time. 24 hours equal one day.
Hour Hand	Shorter hand of the two hands on an analog clock.
Hundredth	One part of 100 equal parts of a whole.
Identity Property **a) Addition** **b) Multiplication**	a) The sum of any number and 0 is that number. b) The product of any number and 1 is that number.
Impossible	Will not happen. Probability of zero.
Improper Fraction	A fraction whose numerator is greater than or equal to its denominator.
In Front Of	A preposition that indicates location of an object.
Inch	A customary unit of measurement of length. 12 inches = 1 foot
Increasing	To grow progressively more (as in size, amount, number, or intensity).
Independent Variable	The variable in a function with the value that is subject to choice.
Inequality	A number sentence that uses < or > symbols.
Infinite	Immeasurably great or large. Boundless.
Integer	The set of positive whole numbers, their opposites, and zero.
Interquartile Range	The range of the middle half of a set of data. The difference between the upper and lower quartiles.
Intersecting Lines	Two separate lines that are not parallel.
Inverse Operations	Operations that undo each other.
Isosceles Triangle	A triangle with at least two sides of the same length.
Join	To put or bring together so as to make continuous or form a unit.
Key	Explanation of what each symbol represents in a pictograph.
Kilogram	A metric unit of mass equal to 1,000 grams.
Kilometer	A metric unit of length equal to 1,000 meters.
Kite	A quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are adjacent to each other.
Least	Smallest.
Least Common Multiple	The smallest number, other than zero, that is a multiple of two or more numbers.
Length	A measurement of distance or dimension. Usually refers to the longer dimension.
Less	The smaller of two numbers being compared.
Less Likely	Not as probable to happen.

Less Than	The smaller of two numbers being compared. (<)
Like Denominators	Denominators in two or more fractions that are the same.
Line	A straight path of points that goes on forever in two directions.
Line of Symmetry	A line on which a figure can be folded into two congruent parts.
Line Plot	A display of responses along a number line with x's recorded above the response to indicate the number of times the response occurred.
Line Segment	Part of a line that has two endpoints.
Liter	A metric unit of capacity equal to 1,000 milliliters.
Long	Having greater length than usual.
Longer	A word used when comparing the length of two objects.
Lowest Terms	Fractions reduced to simplest form.
Magnitude	Greatness of size or amount.
Mass	The amount of matter in an object.
Maximum (Upper Extreme)	The greatest value in a set of numbers.
Mean	The sum of the values in a data set divided by the number of values. The average.
Measure	A number or quantity that records a directly observable value or performance.
Median	The middle value when a set of numbers is listed from least to greatest.
Meter	A metric unit of length equal to 1,000 millimeters.
Metric System	A system using decimals and powers of 10 to measure length, mass, and capacity.
Metric Unit	A unit of measurement which units are based on tens.
Mile	A customary unit of length equal to 5,280 feet.
Milliliter	A metric unit of capacity. 1,000 milliliters equal 1 liter.
Millimeter	A metric unit of length. 1,000 millimeters equal 1 meter.
Minimum (Lower Extreme)	The smallest value in a set of numbers.
Minuend	The quantity from which another quantity, the subtrahend, is to be subtracted.
Minus	To subtract.
Minute	A measure of time. 60 minutes equal 1 hour.
Minute Hand	Longer hand of the two hands on an analog clock.
Mixed Number	A number that has a whole number part and a fractional part.
Mode	The number or numbers that occur most often in a set of data.
Money	Any object or record that is generally accepted as payment for goods and services and repayment of debts.
Month	One of the twelve parts into which a year is divided.
More	The larger of two numbers being compared.
More Likely	More probable to happen.
More than	The larger of two numbers being compared. (<)

Morning	The period of time between dawn and noon.
Multiple	The product of a whole number and any other whole number.
Multiplication	An operation that gives the total number when you put together equal groups. The operation that gives the product of two or more numbers.
Multiplicative Inverse	Two numbers whose product is one.
Multiply	An operation that gives the total number when you put together equal groups. The operation that gives the product of two or more numbers.
Negative Number	Numbers to the left of zero on a number line.
Net	A plane figure pattern, which when folded, makes a solid.
Next To	A preposition that indicates location of an object.
Nickel	A unit of money equal to five cents.
Night	The period of time between the sunset and the sunrise when the sun is below the horizon.
Nonstandard Unit	A number not written with commas separating groups of three digits starting from the right.
Number	A mathematical object used to count, label, and measure.
Number Line	A line that shows numbers in order using a scale.
Numeral	A symbol used to represent a number. (e.g. roman numerals)
Numerator	The number above the fraction bar of a fraction.
Numerical	Of or relating to a number or series of numbers.
Obtuse Angle	An angle that measures greater than 90 degrees and less than 180.
Obtuse Triangle	A triangle in which one angle is an obtuse angle.
O'clock	Phrase used to refer to an hour in time (e.g. 3 o'clock) when the minute hand is straight up.
Octagon	A polygon with eight sides.
Odd	A number not divisible by two.
One-eighth	One part of a whole divided into eight equal parts.
One-fourth	One part of a whole divided into four equal parts.
One-half	One part of a whole divided into two equal parts.
One-sixth	One part of a whole divided into six equal parts.
One-third	One part of a whole divided into three equal parts.
Ones	A single unit or object.
Opposite	The integer on the opposite side of zero from a given number, but at the same distance from zero.
Order of Operations	A set of rules mathematicians use to determine the order in which operations are performed. Work is done in this order: 1) parentheses 2) exponents 3) multiply / divide 4) add / subtract

Ordered Pair	A pair of numbers that gives the coordinates of a point on a grid in this order (horizontal coordinate, vertical coordinate).
Ordinal Numbers	Numbers used to tell the order of people or objects.
Origin	The point where the two axes of the coordinate plane intersect. The origin is represented by the ordered pair (0, 0).
Ounce	Customary unit of capacity or mass.
Outlier	A number very different from the other numbers in a data set.
Pair	Match two similar items that go together.
Parallel (Lines)	(Lines) Always the same distance apart.
Parallelogram	A quadrilateral with two pairs of parallel and congruent sides.
Parentheses	A type of bracket used in mathematics in several different cases.
Past	No longer current.
Pattern	A set of numbers or objects in which all the members are related with each other by a specific rule.
Penny	A unit of money equal to one cent.
Pentagon	A polygon with five sides.
Percent	A ratio where the first term is compared to 100.
Perimeter	The distance around a figure.
Period	A group of 3 digits in a number. Periods are separated by a comma and start from the right of a number.
Perpendicular Lines	Two lines intersecting to form right angles.
Pictograph (Picture Graph)	A chart pictures or symbols to compare data that can be counted.
Pint	A customary unit of capacity equal to two cups.
Place Value	The position of a digit in a number that is used to determine the value of the digit.
Plane	A flat surface that extends forever in all directions.
Plane Shape	The geometric shapes whose surface is flat.
Plot	Locate and mark a point on a coordinate grid using a given ordered pair.
Plus	Another term meaning addition.
PM	Time between noon and midnight.
Polygon	A closed figure formed by a finite number of coplanar segments called sides such that 1) the sides that have a common endpoint are noncollinear, and 2) each side intersects exactly two other sides, but only at the endpoints called vertices.
Point	An exact location in space.
Positive Numbers	Numbers to the right of zero on a number line.
Pound	A customary unit of mass equal to 16 ounces.
Power of Ten	Used in scientific notation.
Present	Being, existing, or occurring at this time or now.
Prime Number	A whole number greater than one that has exactly two factors, itself and one.

Prism	A polyhedron with two congruent and parallel polygon-shaped faces.
Protractor	An instrument used to measure and draw angles.
Product	The result of multiplication.
Pyramid	A polyhedron whose base is a polygon and whose other bases are triangles that share a common vertex.
Quadrant	The four sections of a coordinate grid that are separated by the axes.
Quadrilateral	A four-sided polygon.
Quantity	How much there is or how many there are of something.
Quart	A customary unit of capacity. Four quarts equal one gallon.
Quarter	A unit of money worth 25 cents.
Quarter Inch	One of four equal parts of an inch.
Quarter Of (quarter hour)	The point on a clock's face marking either 15 minutes after or 15 minutes before an hour.
Quartiles	The four sections on a coordinate plane. a) 1^{st} Quadrant: (+x, +y) b) 2^{nd} Quadrant: (-x, +y) c) 3^{rd} Quadrant: (-x, -y) d) 4^{th} Quadrant: (+x, -y)
Quotient	The number other than the remainder that is the result of dividing.
Range	The difference between the greatest and least numbers in a set of data.
Rate	A ratio that compares two quantities with different units of measure.
Ratio	A pair of numbers that shows a comparison of two quantities that can be written as *a*:*b*, *a*/*b*, or *a* to *b*.
Rational Number	Any number that can be written as a quotient *a*/*b*, where *a* and *b* are integers and $b \neq 0$.
Ray	Part of a line with one endpoint, extending forever in only one direction.
Reasonableness	Not excessive or extreme.
Reciprocal	Two numbers whose product is one.
Rectangle	A quadrilateral with four right angles.
Rectangular Prism	A prism with six rectangular faces.
Rectillinear Figure	Figures bounded by straight lines.
Regroup	To name a whole number in a different way.
Remainder	The part that is left over after trying to divide into equal groups.
Rhombus	A parallelogram with all sides the same length.
Right Angle	An angle that measures exactly 90 degrees.
Right Triangle	A triangle in which one angle is a right angle.
Rounding	A process that tells which multiple of 1, 10, 100, 1,000, etc. a number is closest to.

Rule	The pattern used to determine the next number of a sequence.
Ruler	A tool used to measure length.
Scale **a) Measuring device** **b) Unit of measure** **c) Dimensional sizing**	a) An instrument used for weighing. b) A system of marks at fixed intervals used in measurement or graphing. c) The ratio of length used in a drawing, map, or model to the length of the object in reality.
Scalene Triangle	A triangle in which no sides have the same length.
Second	1/60 of a minute.
Sequence	A set of numbers in a specific order.
Shape	A geometrical description of the part of that space occupied by the object.
Shorter	A word used when comparing the length of two objects.
Side	A line segment connected to other segments to form a polygon.
Similar	Figures that have the same shape, but not necessarily the same size.
Simplest Form	A fraction in which the greatest common factor of the numerator and denominator is one.
Size	The physical dimensions, proportions, magnitude, or extent of an object.
Slide (Translation)	The image of a figure that has been moved to a new position without flipping or turning.
Smaller	A word used when comparing the size of two objects.
Solid Figure	A figure that has three dimensions and volume.
Sort	To group or organize according to shared attributes.
Sphere	A solid figure with all points the same distance from the center point.
Spread	To open or expand over a larger area.
Square	A rectangle with all sides the same length.
Square Unit	A measurement used when measuring area.
Standard Unit (Form)	A number written with commas separating groups of three digits starting from the right.
Statistics	Numerical data that have been collected and analyzed.
Strategy	Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.
Substitution	Use algebraic methods to find an exact solution of a system of equations.
Subtract	Take away, remove, or compare.
Subtraction	Operation used to find the difference between two numbers.
Subtrahend	A quantity or number to be subtracted from another.
Sum	The result of addition.

Surface Area	The sum or the areas of each face of a polyhedron.
Symmetrical Figure	A figure that can be folded into two congruent parts that fit on top of each other.
Symmetry **a) Line** **b) Point**	a) A figure has line symmetry when it can be folded along a line so the two halves match exactly. b) A figure has point symmetry when it can be turned exactly 180 degrees about a point and fit exactly on itself.
Table	Used to show how one quantity is related to another.
Taller	A word used when comparing the height of two objects.
Tally Marks	Marks used to show the frequency of data.
Temperature	The degree of hotness or coldness of a body or environment.
Tens	Something with ten parts or ten units.
Tenth	One out of ten equal parts of a whole.
Terms	The two numbers being compared in a ratio.
Thermometer	A tool used to measure temperature.
Thirds	To divide an object into three equal parts.
Three-Dimensional	Existing in three dimensions; having length, width, and height.
Time	The measured or measurable period during which an action, process, or condition exists or continues.
Time Interval	A definite length of time marked off by two instants.
Today	The present day.
Tomorrow	The day following the present day.
Trapezoid	A quadrilateral with exactly two parallel sides. OR A quadrilateral with one pair of parallel sides and one pair of sides that is not parallel.
Triangle	A polygon with three sides.
Triangular Prism	A prism with triangular bases.
Triangular Pyramid	A pyramid with a triangular base.
Turn	To cause to move around an axis or center.
Two-Dimensional	Having length and width.
Unit Fraction	A fraction with a numerator of 1.
Unit Rate	A rate in which the second number in the comparison is one unit.
Unknown	A number we do not know. Represented by the variable in an expression.
Unlike	Not the same.
Unlike Denominators	Denominators in two or more fractions that are different.
Value	An amount, as of goods, services, or money, considered to be a fair and suitable equivalent for something else.
Variable	A letter, such as *n*, that stands for a number in an expression or an equation.

Vertex (vertices)	The point at which two line segments, lines, or rays, meet to form an angle.
Volume (liquid)	A number of cubic units of space a solid figure takes up.
Week	A unit of time equal to seven days.
Weight	A measure of how light or how heavy something is.
Whole	The entire object. 100% of an object.
Whole Numbers	The set {0, 1, 2, 3, …}
Word Form	A number written in words using place value.
Width	A measurement of distance or dimension. Usually refers to the shorter dimension.
Whole	The full quantity, amount, extent, number, etc.
X-axis	On a coordinate grid, the horizontal axis.
X-coordinate	The first number in an ordered pair that tells the position left or right of the y-axis.
Y-axis	On a coordinate grid, the vertical axis.
Y-coordinate	The second number in an ordered pair that tells the position above or below the x-axis.
yard	Customary unit of length equal to three feet.
year	A unit of time equal to 365 days, 52 weeks, or 12 months.
Yesterday	The day prior to the present day.
Zero Property of Multiplication	In multiplication, the product of a number and 0 is 0.

Table 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 = ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2 – 4 of National Research Council (2009, op. cit., pp. 32, 33).

Table 2. Common multiplication and division situations.[7]

	Unknown Product 3 x 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 x ? = 18, and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? x 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example:* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example:* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example:* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example:* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example:* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples. How many rows will there be? *Area example:* A rectangle has 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example:* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example:* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example:* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a x b = ?	a x ? = p, and p + a = ?	? x b = p, and p + b = ?

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

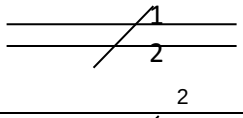
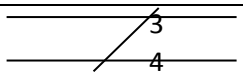
Associative property of addition	(a + b) + c = a + (b + c)
Commutative property of addition	a + b = b + a
Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every a there exists –a so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property of 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every a $\neq$ 0 there exists $1/a$ so that a x $1/a = 1/a$ x a = 1
Distributive property of multiplication over addition	a x (b + c) = a x b + a x c

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a
Symmetric property of equality	If a = b, then b = a
Transitive property of equality	If a – b and b – c, then a = c
Addition property of equality	If a = b, then a + c = b + c
Subtraction property of equality	If a = b, then a – c = b – c
Multiplication property of equality	If a = b, then a x c = b x c
Division property of equality	If a = b and c ≠ 0, then a $\div$ c = b $\div$ c
Substitution property of equality	If a = b, then b may be substituted for a In any expression containing a

Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b
If a > b and b > c, then a > c
If a > b, then b < a
If a > b, then -a < -b
If a > b, then a $\pm$ c > b $\pm$ c
If a > b and c > 0, then a x c > b x c
If a > b and c < 0, then a x c < b x c
If a > b and c > 0, then a + c > b + c
If a > b and c < 0, then a + c < b + c

7 – 12 Mathematics Glossary	
Vocabulary	Definition
Absolute value	The distance of a number from zero on the number line.
Acute angle	An angle that measures less than 90 degrees.
Addend	Any number being added.
Additive inverse	The opposite of a number. When a number is added to its additive inverse, the sum is zero.
Algorithm	A step-by-step method for computing or carrying out any mathematical procedure.
Alternate exterior angles	 2 In the figure, a transversal intersects two lines. ∠1 & ∠2 are alternate exterior angles.
Alternate interior angles	 4 In the figure, a transversal intersects two lines. ∠3 & ∠4 are alternate interior angles.
Amortization Schedule	A table detailing each periodic payment on an amortizing loan (typically a mortgage).
Angle	Two rays that share an endpoint.
Angle bisector	A ray that divides an angle into two congruent angles.
Angle of depression	The angle between the line of sight and the horizontal when an observer looks downward.
Angle of elevation	The angle between the line of sight and the horizontal when an observer looks upward.
Arc	A part of a circle that is defined by two endpoints.
Area	The measure of the interior region of a two-dimensional figure or the surface of a three-dimensional figure.
Arithmetic series (progression)	A set of numbers in which the difference between any two consecutive numbers is the same.
Array	An arrangement of objects in equal rows.
Associative Property	
Attribute	A characteristic.
Axes	A reference line from which distances or angles are measured on a coordinate grid.
Axis of symmetry	A line which divides the graph of an equation into two congruent halves.
Balanced Budget	A situation in financial planning or the budgeting process where total revenues are equal to or greater than total expenses.
Bank Reconciliation	A process that explains the difference between the bank balance shown in an organization's bank statement and the checkbook register.
Bank Statement	A summary of financial transactions which have occurred over a given period of time on a bank account held by a person or business with a financial

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Vocabulary	Definition
	institution.
Bar graph	A graph that uses the height or length of rectangles to compare data.
Base e	The base of the natural logarithms. The irrational number 2.718…
Base ten	A number system in which each place has ten times the value of the next place to its right.
Box (box-and-whisker) plot	A way to display a distribution of data values but using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.
Budget	A plan for managing income and expenses, usually for a set period of time.
Budget Item	Each expense mentioned in a budget.
Capacity	The greatest amount that a container can hold.
Central angle	An angle with the vertex at the center of a circle.
Central Tendency (measures of)	Relates to the way in which quantitative data tend to cluster around some value.
Check Register	A personal record of your checking account.
Chord	A segment with endpoints that are on the circle.
Commission	A payment based on a percentage of total sales.
Coordinate plane (Cartesian)	A two-dimensional system in which a location is described by its distances from two intersecting, usually perpendicular, straight lines called axes.
Causation	An act of bringing about a desired result.
Circle	The locus of all points in a plane equidistant from a given point called the center of the circle.
Circumference	Perimeter of a circle.
Classify	Categorize things or objects.
Cluster	Location of a large grouping of data.
Combination	A group of items or events. Placing these items or events in a different order does not create a new combination.
Common denominator	For two or more fractions, a common denominator is a common multiple of the denominators.
Common fraction	Any fraction whose numerator and denominator are natural numbers (¼, ½, ⅓…).
Common Multiple	A number that is a multiple of two or more numbers.
Commutative property	Changing the order does not change the end result (applies to addition and multiplication).
Compass	An instrument used to draw a circle.
Complementary angles	Two angles with measures that have a sum of 90°.
Complementary events	Two or more mutually exclusive events that together cover all possible outcomes. The sum of the probabilities of complementary

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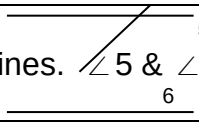
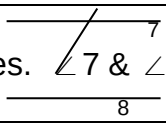
Vocabulary	Definition
	events is 1.
Complex number	A number in the form a + bi where a and b are real numbers and I = $\sqrt{-1}$.
Composite functions	Combination of two functions, where you apply the first function and get an answer, and then fill *that answer* into the second function.
Composite number	A number greater than zero that has more than two different factors
Compound events	A combination of simple events.
Compute	To find a numerical result, usually by adding, subtracting, multiplying or dividing.
Cone	A three-dimensional figure with one curved surface, one flat surface(usually a circle), one curved edge, and one vertex
Congruent	Having exactly the same size and shape.
Conjecture	An educated guess based on know information
Consecutive interior angles	 In the figure, a transversal intersects two lines. $\angle 5$ & $\angle 6$ are consecutive interior angles.
Constant	A quantity that always stays the same.
Contextualized problems	Solving real life situations using mathematics.
Coordinate grid/plane (Cartesian)	A two-dimensional system in which a location is described by its distances from two intersecting, usually perpendicular, straight lines called axes.
Correlation	An association between two variables used in statistics.
Correlation coefficient	A numerical value between -1 and 1 inclusive that measures the strength and direction of a linear relationship between two variables.
Corresponding Angles	 In the figure, a transversal intersects two lines. $\angle 7$ & $\angle 8$ are corresponding angles.
Corresponding parts	Matching parts of congruent polygons.
Cosine	For an acute angle of a right triangle, the ratio of the measure of the leg adjacent to the acute angle to the measure of the hypotenuse.
Credit	Credit is the trust which allows one party to provide resources to another party where that second party does not reimburse the first party immediately.

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Vocabulary	Definition
Credit Card	A credit card is a payment card issued to users as a system of payment. It allows the cardholder to pay for goods and services based on the holder's promise to pay for them.
Counterexample	An example used to show that a given statement is not always true.
Cube	A right rectangular prism with all sides and angles equal.
Cube number	A number to the third power.
Cubic root	A number multiplied by itself twice to get the given number.
Cubic unit	Measurement used when measuring volume.
Currency Exchange	Marketplace used to exchange currencies from multiple countries.
Customary system	A system of measurement used in the United States. The system includes units for measuring length, capacity, weight, and temperature.
Cylinder	A three-dimensional figure with two circular bases that is parallel and congruent.
Data	Information, especially numerical information.
Debit Card	A way of paying for goods that deducts expenses from a cardholder's bank account.
Debt	An amount of money borrowed by one party from another.
Degrees a)angle measurement b) temperature measurement	a) A unit used to measure angles. b) A measurement of hotness or coldness (Celsius, Fahrenheit, Kelvin).
Density	The mass per unit volume.
Dependent events	Two events in which the outcome of the first event affects the outcome of the second event.
Deposit Slip	A small written form that is sometimes used to deposit funds into your account.
Depreciation	The decrease in value.
Digit	Any one of the ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Dilation	A transformation that shrinks or enlarges a figure.
Dimension	Measurement in one direction.
Discount	An amount subtracted from the regular price.
Distance Formula	Formula used to find the distance between two points.
Distributive Property	$a(b + c) = ab + ac$
Dividend	The amount of profit that a shareholder earns for each share of stock.

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Vocabulary	Definition
Divisor	
Domain	In a function, f(x), the possible values for x in the given situation.
Dot/Line plot	A way to display data values where each is shown as a dot or mark above a number line.
Draft	A form of check where the funds are taken directly from the financial institution.
Drawee	The party on which an order for the payment of money is drawn.
Drawor	The party that receives payment from the drawee.
Electronic Deposit	The money is transferred directly to the recipient bank through a payment system.
Electronic Funds Transfer (EFT)	Any transfer of funds from one account to another that occurs electronically.
Empirical Rule	Based on or characterized by observation and experiment instead of theory.
Endorsement	The signature on a check, contract, instrument, or other document.
Equation	A mathematical sentence with an equal sign.
Equivalent	Having the same value.
Estimate	A number close to an exact amount; an estimate tells about how much or about how many.
Euclidean Geometry	A geometrical system in which a plane is a flat surface made up of points that extend infinitely in all directions.
Expenses	Money spent to pay for specific costs.
Experimental probability	A statement of probability based on the results of a series of trials.
Exponential Notation	A way of writing numbers using exponents.
Exponents	The number that tells how many equal factors.
Expression **a) For Algebraic EX: a + b** **b) For Numerical EX: 6+4**	A combination of variables, numbers, and operation symbols that represents a mathematical relationship.
Extreme Value	The maximum and minimum values of a function.
Face	A flat surface on a solid figure.
Fact families	Number sentences that relate addition and subtraction or multiplication and division. Each number sentence in the fact family has the same numbers.
Factor	An integer that divides evenly into another.
Factorial	The product of a whole number and every positive whole number less than itself. Abbreviated n! and say: n factorial 4! = 4 x 3 x 2 x 1 = 24.
Factoring	Rewriting a polynomial expression as a product.

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Vocabulary	Definition
Fixed Rate	An interest rate that does not change over the life of a loan or other form of credit.
Fluently	Efficiently and accurately.
Fraction	A way of describing a part of a whole or a group.
Frequency	The number of cycles in a given unit of time.
Frequency distribution chart	A way to display how often an item, number, or range of numbers occurs.
Function	A relation in which every value of x has a unique value of y.
Fundamental counting principal	If one event can happen in x ways and a second, independent, event can happen in y ways, the two can occur together in a x • y ways.
Generalization	A statement or conclusion that is derived from and applies equally to a number of cases.
Geometric series (progression)	A sequence of terms in which each term is created by multiplying the previous term by a constant.
Greatest Common Divisor	The greatest number that divides into two or more numbers with no remainder.
Gross Earnings	The amount of full earnings before deductions.
Histogram	A bar graph in which the labels for the bars are consecutive groups of numbers.
Horizontal axis	Axis on a coordinate plane that runs from side to side.
Identity property of addition	If you add zero to a number the sum is the same as that number.
Identity property of multiplication	If you multiply a number by one, the product is the same as the original number.
Imaginary number	The square root of a negative real number. It cannot be shown on the number line.
Income	The total money earned.
Independent events	Two events in which the outcome of the first event does not affect the outcome of the second event.
Inductive reasoning	Reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction.
Inequality	A mathematical sentence that compares two amounts using the symbols; >, <, ≤, ≥, or ≠.
Inference	Judge whether the number you found is the number you expected.
Installment Buying	Purchasing a commodity over a period of time. The buyer gains the use of the commodity immediately and then pays for it in periodic payments called installments.
Insurance	A form of risk management primarily used to hedge against the risk of a contingent, uncertain loss.

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Vocabulary	Definition
Integers	Whole numbers and their opposites (…-3, -2, -1, 0, 1, 2, 3…)
Intercept	The point in which a line intersects the x-axis and y-axis.
Intersecting lines	Two separate lines that are not parallel.
Inverse function	A function in which two variables are inversely proportional.
Irrational numbers	Numbers that cannot be written as a ratio of two integers. If you try to write an irrational number as a decimal, the digits never terminate and never repeat. (EX $\sqrt{2}$ = 1.41421356…)
Irregular polygon	A polygon in with all of the sides and angles are not congruent.
Least Common Multiple	The smallest number, greater than zero, found in all the list of multiples of two or more numbers.
Likelihood	The chance of something happening.
Line	A set of connect points continuing without end in both directions.
Line graph	A graph used to show change over time with points connected by line segments.
Line of symmetry	A line that divides a figure into two congruent halves that are mirror images of each other.
Line segment	A part of a line with two endpoints.
Linear equation	An equation in two variables whose graph in a coordinate plane is a straight line.
Logarithm	In the function $x = b^y$, y is called the logarithm, base b, of x.
Logarithmic Function	The function $y = \log_b x$, where $b > 0$, and $b \neq 1$, which is the inverse of the exponential function $y = b^x$.
Magnitude	Size and scale reflected by a value.
Markup	The difference between the cost of a good or service and its selling price.
Maturity Value	The amount of money the issuer will pay the holder of a security at the redemption date, the face value plus any accrued interest.
Maximum of function	A point at which a function attains its greatest value.
Mean (average)	A measure of center in a set of numbers, computed by adding the values in the list and then dividing by the number of values in the list.
Median	The middle number when numbers are arranged from least to greatest. When the set has two middle numbers, the median is the mean of the two middle numbers.
Metric system	A system of measurement which units are based on tens.
Midpoint	The point on a segment exactly halfway between the endpoints of the segment.
Minimum of function	The least value of a function.

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Vocabulary	Definition
Mode	A number that appears most frequently in a set of numbers. There may be, one, more than one, or no mode.
Monomial	The product of constants and variables.
Mortgage	A financed loan on property.
Multiple	The product of a whole number and any other whole number.
Multiplicative inverse	Two numbers whose product is 1 are multiplicative inverses of one another.
Mutual Funds	An investment vehicle that is made up of a pool of funds collected from many investors for the purpose of investing in securities such as stocks, bonds, money market instruments and similar assets.
Mutually exclusive	Two events that cannot occur at the same time.
Natural Log	Logarithms with base *e*, written in x.
Natural numbers	The counting numbers; 1, 2, 3, 4…
Negative exponent	For any real number $a \neq 0$, and any integer n, $a^{-n} = 1/a^n$
Net Earnings	The amount a worker receives after deductions are subtracted from gross pay.
Nonstandard unit	Using common objects for a unit of measure such as a blocks and pencils.
Non-sufficient Funds	A check has been presented for clearance, but the amount written on the check exceeds the available balance in the account.
Non-Euclidean geometry	Shapes and constructions that do not map directly to any n-dimensional system.
Normal distribution	A bell shaped probability distribution. There are as many values less than the mean as there are values greater than the mean.
Number line	A diagram that represents numbers as points on a line.
Number Theory	The study of numbers and the relationships between them.
Numeral	A symbol or set of symbols representing or naming a number.
Obtuse angle	An angle that measures greater than 90 degrees and less than 180.
One to one correspondence	Used to compare two sets in which one element matches one and only one element in the other set.
Operational symbols	Symbols representing the operations of addition, subtraction, multiplication and division.
Order of operations	A set of rules. It tells you the order in which to compute so that you will get the same answer than anyone else will get.
Ordered pair	A pair of numbers that gives the coordinates of a point on a grid in this order (horizontal coordinate, vertical coordinate).
Ordinal number	A whole number that names the position of an object in sequence (first, second, third…).

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Vocabulary	Definition
Outcome	One of the possible events in a probability situation.
Outlier	A piece of numerical data that is much smaller or larger than the rest of the data in a set.
Outstanding Checks	Checks which have been written, but have not yet cleared the bank on which they were drawn.
Overdrafts	A charge resulting from writing a check against a checking account for more money than is in the account.
Parabola	The graph of a quadratic function. It is a symmetric curve.
Parallel	Always the same distance apart.
Parallelogram	A quadrilateral with two pairs of parallel and congruent sides.
Payee	The person who is to receive the stated amount of money on a check, bill, or note.
Payor	The one who must make payment on a promissory note.
Percentile	A division of ordered data into 100 equal parts. About 1% of the data are in each part.
Perfect square	The product of an integer and itself.
Perimeter	The distance around a figure.
Permutation	An ordered arrangement of elements from a set.
Perpendicular	Forming right angles.
Perpendicular Bisector	In a triangle, a line, segment, or ray that passes through the midpoint of a side and is perpendicular to that side.
Phase Shift	A horizontal translation of a trigonometric function.
Pi (π)	An irrational number represented by the ratio of the circumference of a circle to the diameter of the circle. 3.14
Pictograph	A graph that uses pictures or symbols to show data.
Plane	A flat surface that exceeds infinitely in all directions.
Plot	Mark points on a graph.
Point	A basic undefined term of geometry. No actual size. A point is a location.
Point of Tangency	For a line that intersects a circle in only one point, the point at which they intersect.
Point slope form	The equation of a line in the form $y-y_1 = m(x-x_1)$ where m represents the slope of the line and (x_1, y_1) is a known point on the line.
Polygon	A closed figure formed by a finite number of coplanar segments called sides such that 1) the sides that have a common endpoint are noncollinear, and 2) each side intersects exactly two other sides, but only at the endpoints called vertices.
Polyhedron	A solid figure in which all the faces are polygons.
Polynomial	The sum of monomials.
Population	A group of people (or objects or events) that fit a particular description.

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Vocabulary	Definition
Positive number	A number that is greater than zero. Positive numbers are right of zero on a number line.
Postulate	A mathematical statement that is accepted as true without proof.
Powers	The number of times a number is repeated as a factor.
Prediction	A statement of what somebody thinks will happen in the future.
Prime Factorization	The expression of a number as a product of prime factors.
Prime number	A number that has exactly has two different positive factors, itself and 1.
Prism	A three-dimensional figure that has two congruent and parallel faces that are polygons. The rest of the faces are parallelograms.
Probability	The chance of an event happening.
Product	The result of multiplication.
Proof	A logical argument in which each statement you make is supported by a statement that is accepted as true.
Property	A rule about numbers that is always true when you compute no matter which numbers you use.
Proportion	An equation showing that two ratios are equal.
Protractor	An instrument used to measure degrees of an angle.
Pyramid	A polyhedron whose base is a polygon and whose other bases are triangles that share a common vertex.
Pythagorean Theorem	A formula used to find the length of a side of a right triangle, given two sides. $a^2 + b^2 = c^2$
Quadrant	The four sections of a coordinate grid that are separated by the axes.
Quadratic Equation	An equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$.
Quadratic function	A function with a second degree variable (x^2).
Quadrilateral	A four-sided polygon.
Quantitative	Capable of being measured or expressed in numerical terms.
Quantitative change	A relationship that can be expressed in numerical terms.
Quantitative relationships	Numbers that can be expressed or compared in a meaningful way.
Quantity	An amount.
Quartiles	Along with the median, the quartiles divide the ordered set of data into four equal sized groups.
Quotient	
Radius	Any segment with endpoints that are the center of the circle and a point on the circle.

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Vocabulary	Definition
Random Sample	A sample that is chosen without any preference, representative of the entire population
Range **a) For Data** **b) For Function**	a) The difference between the greatest and the least value in a set of data. b) The possible values for y in a function.
Rate	The ratio of two measurements having different units of measure.
Rate of change	The ratio of change in one quantity to the corresponding change in another quantity. (see also slope)
Ratio	A comparison of two numbers or measures using division.
Rational expressions	An algebraic expression that can be written as a fraction whose numerator and denominator are polynomials.
Rational number	A number that can be expressed as a ratio of two integers where the denominator is non-zero.
Ray	A part of a line that has one endpoint and goes on forever in one direction.
Real numbers	The combined set of the rational and irrational numbers.
Rectangle	A quadrilateral with four right angles.
Rectangular prism	A prism with six rectangular faces.
Reflection	A transformation creating a mirror image of a figure on the opposite side of a line.
Regression Line (Line of best fit)	A line, segment, or ray drawn on a scatter plot to estimate the relationship between two sets of data.
Relation	A set of ordered pairs for which all x and y are related in the same way.
Relational symbols	Symbols included are >, <, ≤, ≥, ≠, and =.
Relative position	Determines location of a number when comparing numbers (5 is between 1 and 10 or 6 is less than 8).
Remainder	The part that is left over after trying to divide into equal groups.
Right angle	An angle that measures exactly 90 degrees.
Root **a) exponents** **b) of a function**	a) The inverse of a power. $A^x=b$ or $\sqrt[x]{b}$ = a, a is the x^{th} root of b. b) The value which makes the equation equivalent to zero.
Rotation	A transformation in which a figure is turned a given angle and direction around a point.
Sample	A number of people, objects, or events chosen from a given population to represent the entire group.
Sample space	A list of all possible outcomes of an activity.
Scale **d) Measuring**	a) An instrument used for weighing. b) A system of marks at fixed intervals used in measurement or

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Vocabulary	Definition
device **e) Unit of measure** **f) Dimensional sizing**	graphing. c) The ratio of length used in a drawing, map, or model to the length of the object in reality.
Scatter plot	A graph with one point for each item being measured.
Scientific Notation	A form of writing as the product of a power of ten and a decimal number greater than or equal to one and less than ten.
Sequence	A set of numbers in a specific order.
Side	A line segment connected to other segments to form a polygon.
Signature Card	A card that must be signed by an individual who opens an account at a bank.
Similar (figures)	Figures that have the same shape, but not necessarily the same size.
Sine	For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the hypotenuse.
Sketch	A drawing completed quickly, but still recognizable.
Skewed distribution	Distribution that shows bunching at one end and a long tail stretching out the other direction.
Slope	The measures of steepness of a line as you look at it from left to right. A numerical value for slope is found using two points on the line and dividing the change in y-value by the change in x-value.
Slope intercept form	A form of a linear equation, y=mx + b, where m is the slope of the line and b is the y-intercept.
Solid figure	A figure with three dimensions.
Sphere	A solid figure made up of points that are the same distance from a point called the center.
Square number	A number to the second power.
Square root	The number when multiplied by itself results in a given number.
Square units	A measurement used when measuring area.
Standard deviation	The measure of dispersion equal to the square root of the variance.
Standard form for equations **a) Linear** **b) Quadratic**	a) ax + by = c, where a and b are not both zero b) ax^2 +by +c
Stem-and-leaf plot	A way to organize the numbers in a data set so that the numbers themselves make the display.
Strategies	Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.
Substitution	Use algebraic methods to find an exact solution of a system of

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Vocabulary	Definition
	equations.
Sum	The result of addition.
Supplementary angles	Two angles with measures that have a sum of 180°.
Surface area	The sum of the area of all faces and side surfaces of a three-dimensional figure.
Survey	Used to collect information about a population.
Symbols	Something that represents something else.
Symmetry **c) Line** **d) Point**	c) A figure has line symmetry when it can be folded along a line so the two halves match exactly. d) A figure has point symmetry when it can be turned exactly 180 degrees about a point and fit exactly on itself.
Systems of inequalities	A set of two or more inequalities with the same variables.
System of linear equations	Two or more related linear equations for which you seek a common solution.
Table	An arrangement of information or data into columns and rows or a condensed list.
Tangent	For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the leg adjacent to the acute angle.
Tessellation	A pattern that covers a plane by transforming the same figure or set of figures so that there are no overlapping or empty spaces.
Theorem	A statement or conjecture that can be proven true by undefined terms, definitions, and postulates.
Theoretical probability	Finding the probability of an event without doing an experiment or analyzing data.
Three-dimensional	Existing in three dimensions; having length, width, and height.
Transformation	A rule for moving every point in a plane figure to a new location.
Translation (slide)	A transformation that slides a figure a given distance in a given direction.
Transversal	A line that intersects two or more lines in a plane at different points.
Trapezoid	A quadrilateral with exactly two parallel sides. OR A quadrilateral with one pair of parallel sides and one pair of sides that is not parallel.
Triangle	A polygon with three sides.
Triangular prism	A prism with triangular bases.
Two-dimensional	Having length and width.
Unit	A precisely fixed quantity used for measure.
Variable	A quantity that can have different values.

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Vocabulary	Definition
Variance	A measure of dispersion of data centered about the mean.
Venn Diagram	A diagram that uses circles to show how elements among sets of numbers or objects are related.
Vertex (vertices)	The point at which two line segments, lines, or rays, meet to form an angle.
Vertex of quadratic equation	Highest or lowest point
Vertical axis	Axis on a coordinate plane that runs up and down.
Volume	A number of cubic units of space a solid figure takes up.
Whole number	Any of the numbers 0, 1,2,3,4 and so on.
x-axis	On a coordinate grid, the horizontal axis.
x-intercept	A value of x in an ordered pair describing the point at which a line or the graph of a function intersects the x-axis.
y-axis	On a coordinate grid, the vertical axis.
y-intercept	A value of y in an ordered pair describing the point at which a line or the graph of a function intersects the y-axis.
Zeros of a function	Values of the variable for which the value of a function is zero. Also called roots of a function.

Table 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 = ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2 – 4 of National Research Council (2009, op. cit., pp. 32, 33).

Table 2. Common multiplication and division situations.[7]

	Unknown Product 3 x 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 x ? = 18, and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? x 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example:* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example:* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example:* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example:* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example:* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples. How many rows will there be? *Area example:* A rectangle has 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example:* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example:* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example:* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a x b = ?	a x ? = p, and p + a = ?	? x b = p, and p + b = ?

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	(a + b) + c = a + (b + c)
Commutative property of addition	a + b = b + a
Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every a there exists –a so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property of 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every a $\neq$ 0 there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over addition	a x (b + c) = a x b + a x c

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a
Symmetric property of equality	If a = b, then b = a
Transitive property of equality	If a – b and b – c, then a = c
Addition property of equality	If a = b, then a + c = b + c
Subtraction property of equality	If a = b, then a – c = b – c
Multiplication property of equality	If a = b, then a x c = b x c
Division property of equality	If a = b and c ≠ 0, then a $\div$ c = b $\div$ c
Substitution property of equality	If a = b, then b may be substituted for a In any expression containing a

Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b
If a > b and b > c, then a > c
If a > b, then b < a
If a > b, then -a < -b
If a > b, then a $\pm$ c > b $\pm$ c
If a > b and c > 0, then a x c > b x c
If a > b and c < 0, then a x c < b x c
If a > b and c > 0, then a + c > b + c
If a > b and c < 0, then a + c < b + c

Assessment and Evaluation

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